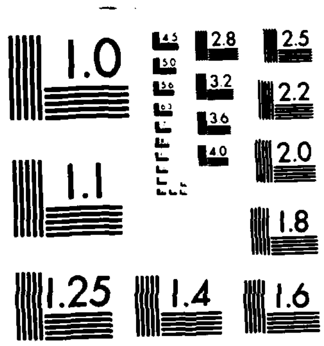
AD-A127 043 AERODYNAMICS OF AIRFOILS SUBJECT TO THREE-DIMENSIONAL PERIODIC GUSTS(U) NOTRE DAME UNIV IN DEPT OF AEROSPACE AND MECHANICAL ENGINEERI.. H ATASSI JUL 82 1982-10 1/1

UNCLASSIFIED AFOSR-TR-83-0231 AFOSR-81-0084 F/G 1/1 NL

END
DATE
FILMED
5 -83
DTIC


1.0
2.8
2.5
3.2
2.2
3.6
4.0
2.0
1.1
1.8
1.25
1.4
1.6

AFOSR-TR- 83-0231

FINAL REPORT

TO THE

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH

ON CONTRACT NO. 81-0084 ENTITLED

AERODYNAMICS OF AIRFOILS SUBJECT TO THREE-DIMENSIONAL PERIODIC GUSTS

Aerodynamics Laboratory

Department of Aerospace
and Mechanical Engineering

University of Notre Dame
Notre Dame, Indiana 46556

ation Division
1982-10

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AFOSR-TR- 83-0231	2. GOVT ACCESSION NO. AD-A127 041	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) AERODYNAMICS OF AIRFOILS SUBJECT TO THREE-DIMENSIONAL PERIODIC GUSTS		5. TYPE OF REPORT & PERIOD COVERED FINAL 1 Jan 81 - 30 Jun 82
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) H ATASSI		8. CONTRACT OR GRANT NUMBER(s) AFOSR 81-0084
9. PERFORMING ORGANIZATION NAME AND ADDRESS UNIVERSITY OF NOTRE DAME DEPT OF AEROSPACE & MECHANICAL ENGINEERING NOTRE DAME, IN 46556		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 61102F 2307/A4
11. CONTROLLING OFFICE NAME AND ADDRESS AIR FORCE OFFICE OF SCIENTIFIC RESEARCH/NA BOLLING AFB, DC 20332		12. REPORT DATE July 1982
		13. NUMBER OF PAGES 64
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

Approved for Public Release; Distribution Unlimited.

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

UNSTEADY FLOW
PERIODIC GUSTS
THREE-DIMENSIONAL GUST

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report outlines our research activities for the period of the AFOSR grant. It is not intended to give a detailed description of our work. Such details are described in our publications and cited references. The report, however, defines the scope of our research on the unsteady aerodynamics and the stability analysis of turbomachine components and its relevance to ongoing technological developments in turbomachine design. The mean topic of our research is the unsteady aerodynamics of lifting airfoils subject to three-dimensional gusts. However, the mathematical methods we have developed for this

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

83 04 20 214

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) purpose, evolved into a general theory for unsteady vortical and entropic distortions of potential flows. This theory can be also applied within the approximation of the rapid distortion theory of turbulence to study the change in turbulence characteristics near the stagnation point of a bluff body. The mathematical theory was applied to study the aerodynamics of lifting airfoils subject to three-dimensional gusts. For low Mach numbers, an integral equation was derived and a numerical scheme was developed for its solution. All previous gust analyses correspond to particular cases of our present work. We have therefore compared our results with previous linear and nonlinear theories. These results show significant three-dimensional and nonlinear effects. Finally, we have carried out a comparison between our previously developed theory for highly loaded cascades with data from experiments being conducted by Frank Carta at United Technologies Research Center. The results show excellent agreement between our analysis and the experimental data.

11

FINAL REPORT

TO THE

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH

ON CONTRACT NO. 81-0084 ENTITLED

AERODYNAMICS OF AIRFOILS SUBJECT TO THREE-DIMENSIONAL PERIODIC GUSTS

FOR THE PERIOD OF JANUARY 1, 1981 TO JULY 31, 1982

BY

H. ATASSI

PROFESSOR

DEPARTMENT OF AEROSPACE AND MECHANICAL ENGINEERING

UNIVERSITY OF NOTRE DAME

1982-10

TABLE OF CONTENTS

A

ABSTRACT

This report outlines our research activities for the period of the AFOSR grant. It is not intended to give a detailed description of our work. Such details are described in our publications and cited references. The report, however, defines the scope of our research on the unsteady aerodynamics and the stability analysis of turbomachine components and its relevance to ongoing technological developments in turbomachine design.

The main topic of our research is the unsteady aerodynamics of lifting airfoils subject to three-dimensional gusts. However, the mathematical methods we have developed for this purpose, evolved into a general theory for unsteady vortical and entropic distortions of potential flows. This theory can be also applied within the approximation of the rapid distortion theory of turbulence to study the change in turbulence characteristics near the stagnation point of a bluff body.

The mathematical theory was applied to study the aerodynamics of lifting airfoils subject to three-dimensional gusts. For low Mach numbers, an integral equation was derived and a numerical scheme was developed for its solution. All previous gust analyses correspond to particular cases of our present work. We have therefore compared our results with previous linear and nonlinear theories. These results show significant three-dimensional and nonlinear effects.

Finally, we have carried out a comparison between our previously developed theory for highly loaded cascades with data from experiments being conducted by Frank Carta at United Technologies Research Center. The results show excellent agreement between our analysis and the experimental data.

I. INTRODUCTION

The present research is mainly motivated by the ongoing technological developments in turbomachine blading systems. The new design trends in engine technology tend toward increased thrust per unit engine weight, more fuel efficiency, and more engine stability at various operating conditions. The performance of the engines is greatly enhanced by higher blade loading and increased flow speed. This creates many new problems in fluid mechanics, aerodynamics, structural design, and aeroelasticity.

Our research relates to the stability of the engine components such as blades and guide vanes. These structural components are subject to heavy aerodynamic loads and produce large air turning inside the engine from one stage row to another. In addition to those desired high aerodynamic loads acting upon rotor and stator blades, there are unwanted fluctuating loads that produce flutter and forced vibrations of turbomachinery blades.

Indeed, flows in turbomachines are highly nonuniform. Their irregular patterns initially result from inlet distortion and inlet turbulence as they enter the engine. Subsequently flow interaction with rotors and stators produces more irregular flow patterns characteristic of each stage of the machine. For example, Figure 1 shows a schematic representation of flow behind a rotor blade row. We notice the formation of secondary flows migrating outwardly from the hub, the development of blade surface viscous boundary layers which extends behind each blade, and the formation of tip and hub swirling vortices.

Figure 2 shows a stator blade in the wake of a rotor. The velocity profile behind the rotor has periodic irregularities due to viscous wakes and swirling vortices particularly at the tip. Because of the periodic nature of a rotor blade row, it is proper to assume that the velocity defect and the swirling vortices have essentially the same periodic structure as the rotor itself. This irregular but periodic flow pattern is moving with a velocity U_R with respect to a stator blade. The stator blades are hence subject to a vortical wave called a gust. This phenomenon was first noticed by Kemp and Sears [1]. Usually, the irregularities in the flow are small compared to the mean flow velocity. That is, if U_d is a characteristic of the velocity defect and U_∞ is the mean flow velocity, then $U_d/U_\infty << 1$. Thus, the gust is a vortical wave essentially convected by the mean flow.

Figure 3 shows a schematic decomposition of the gust into longitudinal (chordwise) and transverse components. There is also a spanwise component which is perpendicular to the plane of the figure. The three components of the gust are all significant and for a lifting airfoil (loaded blade) they are strongly coupled together by the mean flow around the airfoil as we have shown recently in [2].

The main objective of the present research is to formulate an aerodynamic theory for a three-dimensional gust interacting with loaded airfoils, and to develop mathematical and numerical procedures to calculate the fluctuating pressure and forces acting upon those airfoils.

Although our work is motivated by turbomachine technology, we should note that our research is of a basic nature and aims at advancing and extending our knowledge and understanding of unsteady flows as they interact with airfoils and blades.

II. GENERAL OUTLINE OF OUR RESEARCH ACTIVITIES

The primary objective of the present research is to calculate the unsteady forces acting upon lifting airfoils subject to three-dimensional gusts. However as we formulated the mathematical problem we found as a spin-off that the theory can be extended to a general splitting theorem for the velocity field of unsteady vortical and entropic distortions of potential flows.

Hence we developed a general mathematical theory for studying the interaction of non-uniform unsteady flows with bodies. A particular application of the theory is the problem of a three-dimensional gust interacting with an airfoil or a cascade of airfoils. Other important applications can be found in the field of aeroacoustics and fan and helicopter noise. Our theory can also be readily applied under certain conditions, to investigate the change in turbulence characteristics particularly near stagnation points and its effects on boundary layer stability.

We are applying our mathematical theory to essentially two problems. First we studied the three-dimensional gust past an airfoil with camber, thickness and mean-flow incidence. Second, we are presently investigating the change in turbulence characteristics near the stagnation point of a bluff body and an airfoil.

Finally, we have carried out a comparison of our previously developed theory of an oscillating cascade of highly loaded blades with the ongoing experimental results of Frank Carta at United Technologies Research Center.

III. UNIFORMLY VALID SPLITTING OF UNSTEADY VORTICAL AND ENTROPIC DISTORTIONS OF POTENTIAL FLOWS

In many aerodynamic problems one deals with nearly inviscid large Reynolds number flows past solid bodies. For streamlined bodies such as airfoils, the flow is well approximated by a potential flow except of course in the boundary-layer and in the limited region where separation occurs. For bluff bodies, the separated region extends into the wake and the potential flow is only valid outside the wake. The problem we are concerned with here is when small disturbances are added upstream which will produce distortion in the flow field. Two important cases arise

(i) Large Structure Disturbances

This is the case where the length ℓ' and the velocity u' defining the scale of the disturbance, and the length ℓ and the velocity U_∞ characterizing the flow upstream are such that

$$\ell'/\ell >>1 \text{ or } 0\ (1)$$

$$\frac{1}{Re} << \left(\frac{u'}{U_\infty}\right) << 1 \ ,$$

where $Re = U_\infty \ell/\nu$ is the Reynolds number. This is the case of disturbances produced in turbomachines by rotor-stator interaction, and in general the case referred to by aerodynamicists as the gust problem.

(ii) Small Structure Disturbances

This is the case where

$$Re^{-\frac{1}{2}} << \frac{\ell'}{\ell} << 1 \quad \text{and} \quad \frac{1}{Re} << \frac{u'}{U_\infty} << \frac{\ell'}{\ell} \ .$$

This is the case of small structure turbulence interacting with a body. However, the integral scale ℓ' is assumed larger than the boundary layer thickness so that no direct coupling between the incident disturbance and the viscous boundary-layer will occur.

In both cases a linear inviscid theory can be developed to model the flow.

If the body is a flat plate at zero incidence, then there is a complete uncoupling between the mean flow and the disturbance. The mathematical treatment reduces to a Laplace equation for an incompressible flow and was carried out first by Sears [3]. For two-dimensional compressible flows and oblique gusts the problem can be reduced to solving a Helmholtz equation as shown by Graham [4]. For a cascade, an integral equation formulation was first developed by Lane and Friedmann [5]. Whitehead [6] gave detailed numerical calculations for arbitrary interblade phase angle. Atassi and Hamad [7] extended the cascade theory to three-dimensional disturbances in subsonic flows.

For a lifting airfoil the coupling between the mean flow and the disturbance is very strong as shown by Goldstein and Atassi [8] for a two-dimensional gust and by Atassi [2] for a three-dimensional gust.

For a bluff body, Hunt [9] treated the interaction of incident disturbance with a circular cylinder by generalizing the "rapid distortion theory of turbulence" developed by Batchelor and Proudman [10] and Ribner and Tucker [11]. These theories predict changes occuring in turbulent flows that are distorted by solid obstacles.

Let us review the mathematical methods developed to deal with this general class of unsteady vortical disturbances.

1. Traditional Splitting of the Velocity Field

In this case the unsteady velocity $\vec{u}$ is split into a solenoidal component $\vec{u}_s$ and an irrotational component $\nabla\phi$

$$\vec{u} = \vec{u}_s + \nabla\phi. \tag{1}$$

Introduce the vector $\vec{A}$ such that $\nabla\cdot\vec{A} = 0$ and such that

$$\vec{u}_s = \nabla \times \vec{A} \quad . \tag{2}$$

This leads to three Poisson equations

$$\nabla^2\vec{A} = -\vec{\omega} \tag{3}$$

where $\vec{\omega}$ is the vorticity which can be determined using Cauchy's solution *[12,p. 276] which depends* on the Lagrangian coordinates of a fluid particle.

The function ϕ satisfies the equation

$$L(\phi) \equiv \frac{D_o}{Dt}\left(\frac{1}{c_o^2}\frac{D_o\phi}{Dt}\right) - \frac{1}{\rho_o}\nabla\cdot(\rho_o\nabla\phi) = \frac{\nabla\cdot(\rho_o\vec{u}_s)}{\rho_o} \quad . \tag{4}$$

The operator L defined in (4) is a second-order linear convective wave operator with non-constant coefficients. Note that

$$\frac{D_o}{Dt} \equiv \frac{\partial}{\partial t} + \vec{U}_o\cdot\nabla$$

is the linearized material derivative, and ρ_o, c_o represent the density and speed of sound of the mean flow field, respectively.

This method leads to three Poisson's equations and an inhomogeneous wave equation (4). For an incompressible flow (4) reduces to Laplace equation.

2. Goldstein's Splitting of the Velocity Field

Instead of splitting the velocity field into solenoidal and irrotational fields, Goldstein [13] splits $\vec{u}$ into a known vortical solution

$$u_i^{(H)} = A_j (\vec{a},o) \frac{\partial a_j}{\partial x_i} \qquad (5)$$

and an irrotational field $\nabla\phi$ satisfying

$$L(\phi) = \frac{\nabla \cdot (\rho_o \vec{u}^H)}{\rho_o} \quad . \qquad (6)$$

Here $\vec{a}$ represents the Lagrangian coordinates of a fluid particle and A_j are the disturbance components upstream.

The advantage of Goldstein's splitting lies in the fact that the mathematical problem has been reduced to solving equation (6). So instead of four partial differential equations we have only one single equation.

The method has been applied successfully by Goldstein and Durbin [14] to flows with no stagnation points.

3. Flows with Stagnation Points

Practically, all flows past airfoils and obstacles have a stagnation point on the body. In this case the Lagrangian coordinates used in both the traditional and the Goldstein's splittings have a singularity. As a result the vorticity $\vec{\omega}$ and the vortical velocity $\vec{u}^{(H)}$ are both singular along all surfaces of the body and its wake. In fact we have proved the following result:

> "For a flow past a body with a stagnation point, both the traditional and Goldstein's splittings lead to a singular and indeterminate vortical solution along the body surface. The boundary-value problem for the potential part of the disturbance would then have to satisfy singular and indeterminate Neumann's boundary conditions at the body surface."

This singular and indeterminate behavior all along the surface of the body is a major difficulty for analytical and numerical solutions to the gust problem and to the analysis of turbulent flows. In fact, we can even state that unless the solution is regularized, the treatment will be incorrect.

In order to illustrate the effect of the stagnation point, we consider the airfoil shown in Fig. 4. The mean flow streamlines are also shown. Figures 5 to 12 show the streamwise and normal components of the vorticity and the vortical velocity for an initial harmonic normalized disturbance at 45° to the mean flow. The reduced frequency $k_1 = 4$. The choice of a large reduced frequency is made to illustrate the full variation of the vorticity and vortical velocities over the airfoil surface. Because of the singular behavior at the airfoil surface, we only considered the streamlines $\psi = \pm 0.01$. The abscissa s is the length of the arc along the airfoil. The origin $s = 0$ correpsonds to the stagnation point location, and the other two vertical lines correspond to the trailing edge on the pressure side and the suction side. The amplification of the streamwise component of the vorticity and the normal component of the vortical velocity are quite marked. They will become, of course, infinite for $\psi = 0$ as we move closer to the surface of the ai ,oil. The phase of all the components will be indeterminate at the airfoil surface.

Hence, if one uses the traditional method of solution, Eqs. (3,4) will have singular and indeterminate inhomogeneous terms. On the other hand if one uses Goldstein's method, the vortical solution $\vec{u}^H$ is singular and indeterminate along the surface of the airfoil and Eq. (6) will have to satisfy singular and indeterminate boundary conditions all along the airfoil surface and its wake.

4. Uniformly Valid Splitting of the Velocity Field

Our mathematical procedure can essentially be summarized by the following theorem :

"The velocity field

$$\vec{u}^{(R)} = \vec{u}^{(H)} + \nabla\tilde{\phi}$$

where $\tilde{\phi}$ satisfies the equation

$$\frac{D_0}{Dt}\tilde{\phi} = 0,$$

and the boundary condition

$$\vec{u}^{H} + \frac{\partial\tilde{\phi}}{\partial n} = 0, \text{ on Airfoil and Wake,}$$

is regular and has the remarkable property that its streamwise and normal components vanish at the body surface and its wake. The singular and indeterminate character of $\vec{u}^{(H)}$ is eliminated."

Hence we propose the following mathematical method for all problems involving vortical and entropic distortions of potential flows.

$$\vec{u} = \vec{u}^{(R)} + \nabla\phi \quad , \tag{7}$$

where

$$\frac{D_0}{Dt}\vec{u}^{(R)} + \vec{u}^{(R)} \cdot \nabla\vec{U} = 0 \tag{8}$$

With homogeneous conditions for $\vec{u}^{(R)}$ along the tangent and the principal normal of the body surface and its wake if any, and

$$L(\phi) = \frac{1}{\rho_0} \nabla \cdot \rho_0 \vec{u}^{(R)} \tag{9}$$

with the conditions

$$\frac{\partial \phi}{\partial n} = 0 \text{ along body surface and}$$

$$\nabla \phi + \vec{u}^{(R)} = \text{prescribed upstream conditions.}$$

More details about this splitting were given in [15] and will be soon published in [16].

General expressions for $\vec{u}(R)$ were also derived for flows that exist for all times for three-dimensional disturbances past two-dimensional and axisymmetric bodies.

5. Incompressible Flows

For an incompressible flow Eq. (9) reduces to

$$\nabla^2 \phi = - \nabla \cdot \vec{u}^{(R)} \tag{10}$$

and hence the problem reduces simply to solving a Poisson's equation.

For a two-dimensional body, it is possible to consider a Fourier expansion spanwise, and the preceding equation reduces to

$$(\nabla^2 + k_3^2)\, \phi = - \nabla \cdot \vec{u}^{(R)} \tag{11}$$

where ∇^2 is the 2-D Laplace operator and k_3 is the wave number in the span direction. This equation is called a Klein-Gordon equation [18] and usually it occurs in studies of oscillations in resisting media. Thus, one should expect a smaller fluctuating pressure for three-dimensional oblique gust than for two-dimensional gusts. These results will be further discussed in our study of the gust problem.

IV. LIFTING AIRFOILS SUBJECT TO THREE-DIMENSIONAL PERIODIC GUST

The gust problem is illustrated in Figure 2. The disturbance as seen by a blade can be expanded in a Fourier series. Therefore without loss of generality, we can treat the case of a harmonic disturbance of the form

$$\vec{u}_\infty = \vec{A}\, e^{i[\vec{k}\vec{x} - U_\infty t]} \qquad (12)$$

The general theory can be developed as an application to our splitting theorem and the mathematical problem is reduced to finding the regular vortical velocity $\vec{u}^{(R)}$, and then solving equation (9) subject to upstream condition (12) and homogeneous condition along the airfoil surface. In addition one should account for the effect of vortex shedding in the wake which here will be represented by a vortex sheet.

The early treatments of the gust problem dealt, of course, with flat plate airfoils. These are the so-called linearized theories which we briefly review here since they will be used for comparison with our present work.

1. Linear Theories for Airfoils and Cascades in Nonuniform Motions

The aerodynamics of flat plate airfoils subject to a gust have been extensively developed at all flow regimes. They constitute an important reference case for our present work, and our experience in linear unsteady aerodynamics is of great help for pursuing the present nonlinear problem.

Although the linear treatment brings about considerable simplification of the mathematical analysis, the only closed form analytical solution for the fluctuating lift and moment is that of Sears [3] for the flat plate in incompressible flow subject to a transverse gust. For compressible subsonic flows, an integral equation is commonly used in the analysis. For a single airfoil we have an equation derived by Possio [18], and for a cascade the treatment was first made by Lane and Friedman [5]. Detailed numerical methods were given by Whitehead [6,19], and numerical codes are now commonly used for two-dimensional gusts interacting with flat plates and cascades.

The case of an obliquely propagating gust interacting with a flat plate would be the linear counterpart of the three-dimensional gust interacting with a lifting airfoil we propose to investigate. This is because the third longitudinal component of the gust does not induce any fluctuating forces when the lift coefficient of the airfoil is zero. Filotas [20], Mugridge [21], and Graham [22] using different formulations studied the response function of a flat plate to an oblique gust.

For a cascade of flat plate airfoils, Atassi and Hamad [7] derived similarity rules for three-dimensional gusts in compressible flows. Atassi and Hamad also developed a computer code for the aerodynamics of swirling flows interacting with cascades and applied their results to determine the level of noise generated by tip vortices.

The most significant features of linear theories are given below:

(i) Single Flat Plate Airfoil

The unsteady lift depends on the two wave numbers k_1 in the flow direction and k_3 in the span direction. Note that k_1 is also the reduced frequency. Figure 13 shows the real and imaginary parts of the lift coefficient L for the spanwise wave number k_3 = 0.0, 0.2, 0.4, 0.6, 1.0, 2.0, and 3.0. The reduced frequency is varied along each curve from 0 to 5. The Mach number is 0.8. The wave number k_2 in the transverse direction was factored out since in the linear case it does not affect the solution.

Figure 13 underscores the importance as well as the complexity of the three-dimensional effects. The essential features of the 3-D gust are:

- At low reduced frequency, the unsteady lift increases as the mean flow Mach number increases.
- The obliqueness of the gust significantly reduces the magnitude of the unsteady lift at low reduced frequency.
- The lift function does not decrease with increasing k_3 when $k_1 \sim k_3$.

(ii) Cascade of Flat Plate Airfoils

The cascade introduces a large number of parameters such as the spacing d/c, the stagger angle χ, and the interblade phase angle which depends on the ratio of the wave length of the disturbance to the cascade spacing.

The essential features of the 3-D gust interacting with a linear cascade are:

- The resonance effect that is the conditions at which the unsteady lift vanishes. This effect occurs even at very large spacing as shown in Figure 14. As the Mach number increases, the cascade lift suddenly collapses while that of a single airfoil is not affected.
- The cascade effect is more pronounced at large Mach number. This effect is illustrated in Figure 15 showing the unsteady pressure for an airfoil and a cascade at M = 0.8.

2. Nonlinear Theory for a 2-D Gust

Figure 16 shows schematically a lifting airfoil subject to a transverse and longitudinal gust. For this problem the important parameters are the reduced frequency k_1, the transverse wave number k_2, as well as the mean flow round the airfoil. The most interesting physical feature of this problem is the coupling between the mean flow and the oncoming vortical disturbance. This coupling distorts the wave length of the gust as it interacts with the airfoil. The theory was developed by Goldstein and Atassi [8], and more detailed results for cambered airfoils at finite incidence to the mean flow were given by Atassi [23,24].

Figure 17 shows the variation of the real and imaginary parts of the lift function L for three different airfoils having the same mean loading. The reduced frequency is varied from 0 to 10, and the gust angle was taken to be 45° ($k_1 = k_2$). The magnitude F of the ratio of the unsteady lift acting on the airfoils of Figure 17, to the Sears function is plotted in Figure 18 versus the reduced frequency. This clearly shows the significant nonlinear effects. One can single out the following nonlinear effects on the gust response function:

(i) The coupling between the mean potential flow and the unsteady oncoming vortical disturbance depends in a complex manner on the meanflow incidence, the geometry of the airfoil and the gust angle. It is not possible to characterize these effects by a simple parameter such as loading. This is clearly seen in Figures 17 and 18 where we took three airfoils having the same loading.

(ii) The effect of the longitudinal gust is strongest at low reduced frequency. The unsteady lift is significantly reduced because of the airfoil loading.

(iii) At high reduced frequency, the response function does not decay but seems to tend toward a periodic function of finite amplitude.

3. Nonlinear Theory for a 3-D Incompressible Gust

For an incompressible flow Eq. (9) reduces to

$$\nabla^2\phi = - \nabla\cdot\vec{u}^{(R)} \tag{13}$$

and hence the problem reduces simply to solving Poisson's equation. Note however that because of vortex shedding in the wake, the solution will have a discontinuity $\Delta\phi$, whose expression we have derived in [2].

In our review of linear theories we noted that an integral formulation has been used for a single airfoil and a cascade. The numerical calculations can be significantly simplified by transforming the differential governing equations into singular integral equations. In fact for a cascade of oscillaitng airfoils, we have used such an integral formulation in [25-29].

For the present problem Eq. (13) can also be transformed into an integral equation. Details will be given in [30], and lead to

$$\phi = - \frac{1}{4\pi} \iiint_R (\vec{u}^{(R)} - \nabla\bar{\phi}_\infty) \nabla G \, dV$$

$$- \frac{1}{4\pi} \iint_B \phi \frac{\partial G}{\partial n} dS + \frac{1}{2\pi} \iint_W \Delta\phi \frac{\partial G}{\partial n} dS_W , \tag{14}$$

where $\vec{u}^{(R)}$ and $\bar{\phi}_\infty$ are functions determined using our splitting theorem and G is a Green's function. The two surface integrals in (14) are over the airfoil (cascade) surface B and the wake sheet W.

Note that if we can determine the Green's function G for (13) such that

$$\frac{\partial G}{\partial n} = 0 \quad \text{along B}$$

then ϕ is given in terms of surface and volume integrals of already known functions.

The first problem and certainly the easiest to which the present theory is applied is the case of 3-D gust acting upon an airfoil of infinite span. In this case, the potential function ϕ of the mean velocity and the streamfunction ψ can be readily determined. We consider Joukowski airfoils in the present study. This class of airfoils is very popular and the expressions for ϕ and ψ can be obtained in closed-form analytically.

First we have to construct our regular vortical solution $\vec{u}^{(R)}$. We take for Lagrangian coordinate $\vec{a}$ Lighthill's drift function Δ, the streamfunction ψ and the ordinate z in the span direction. Δ was defined in [31] and its difference between two points on a streamline is equal to the time it takes a fluid particle to traverse the distance between those points.

(i) Numerical Scheme

In the present case, the integral equation (14) reduces to the form

$$\mathcal{L}\,\phi - \pi\phi = H \tag{15}$$

where the operator

$$\mathcal{L}\,\phi(\vec{x}) \equiv \int_A \phi \,\frac{\partial K_{o}(r)}{\partial n}\, dS, \tag{16}$$

where $r = |\vec{x} - \vec{x}_0|$, $dS = |d\vec{x}_0|$, K_0 is the modified Bessel function of order zero, and $\frac{\partial}{\partial n}$ is the derivative in the normal function to the airfoil surface. The contour integral is taken along the airfoil A. The inhomogeneous term H contains a double integral and a line integral of known functions and depends on an arbitrary constant.

Two conditions will be imposed on the solution:

- The solution should satisfy Kelvin's theorem.
- The solution should satisfy the Kutta condition.

The first condition can easily be satisfied by imposing the requirement that ϕ is continuous at the trailing edge. The second condition means that the pressure is continuous at the trailing edge, it can be written as

$$\mathcal{K}\ (\phi) \equiv \Delta[ik_1\phi + |\vec{U}_0|\ \frac{\partial\phi}{\partial s}] - C_v = 0 \qquad (17)$$

where C_v is a constant depending on the vortical solution of the gust wave.

As we examine Eq. (15), we see that the homogeneous equation

$$\mathcal{L}\ (\phi)\ - \pi\phi\ = 0 \qquad (18)$$

has an eigenfunction ϕ_0 *corresponding* to the eigenvalue π of the operator $\mathcal{L}$, and hence the solution to (15) is not unique. However, the solution will be unique by imposition of condition (17). From the practical point of view, a discretization of (15) will lead to a singular system of linear algebraic equations. If we relax one of these equations and replace it by (17), the resulting system will still be close to singular, and very difficult to solve with precision. This situation is common when one attempts to solve equations where the desired solution is multivalued to account for the lift. However, usually the equation is Laplace's equation, and instead of K_0 we have ℓnr. In this case the eigenfunction ϕ_0 corresponding to π is a constant. In [27], we were able to use this property to solve the system of linear equations. Here a more elaborate scheme is required.

1. Solve the homogeneous eigenvalue for the Hermitian operator. Let $\overset{*}{\psi}_0$ be such an eignefunction.

2. Solve the modified integral equation.

$$(\mathcal{L} - \psi^{*}K)\phi = H \tag{19}$$

It can be shown that (19) is equivalent to (16) and (17), and it has a unique solution. This integral equation has been solved by collocation. The airfoil contour has been divided into N sections and the value of the potential function has been assumed to be constant over each section. Then the requirement for the integral equation to be satisfied in a set of N mid-section points gives us the system of N linear algebraic equations for N unknown values of the potential function. Also numerical representation of the Kutta condition is needed. This has been done by replacing the value of the derivative of the potential function of the trailing edge by a ratio of two finite differences. Such approximation requires closer spacing of the collocation points at the trailing edge region. Similar spacing has been applied also for the leading edge region where potential function gradients are unusually large.

The unsteady pressure along the airfoil surface is given by

$$P' - P'_{TE} = - \rho \frac{D_0}{Dt} (\phi)$$

This expression contains the derivative of the potential function. However, the potential function is known only in a discrete set of points of the airfoil surface. Finite difference behavior for the derivative approximation cannot be used, because it leads to discontinuous pressure distribution. Therefore spline interpolation of the potential function has been made which gives continuous derivatives along the airfoil surface. The first calculations showed that for the third-order polynomial spline much more collocation points are needed to insure considerable smoothness of the pressure distribution for the desired accuracy. For that reason the exponential spline has been used instead of the polynomical spline.

(ii) Results

As an application of our theory we have chosen a typical Joukowski airfoil with camber, thickness, and incidence to the mean flow. The airfoil geometry is shown in Figure 4. It has 0.1 camber, 0.1 thickness, and is placed at 10° incidence to the mean flow. This airfoil is subject to a sinusoidal gust characterized by three wave numbers, the reduced frequency, $k_1 = 1.0$, $k_2 = 1.0$, and $k_3 = 1.0$

Drift Function

The drift function Δ was used as a Lagrangian coordinate along a streamline. This enabled us to integrate the vorticity equation. However, Δ which represents the time increment as we move along a streamline, is rather a complicated function of the fluid particle position. Figure 19 shows a three-dimensional plot of $D = \Delta - \phi_0$, where ϕ_0 is the mean potential velocity function versus the coordinate x and the streamfunction ψ . The lines stretching from left to right correspond to constant x, while those stretching in the x-direction correspond to constant ψ (along the same mean streamline). Note that far from the airfoil, located between -1 and +1, there are two plateaux for D at different levels because of the circulation round the airfoil. At streamlilnes closer to the suction side of the airfoil there is a significant decrease in the value of D until we get very close to the airfoil, then the effect of the stagnation point begins to be felt and as a result D increases sharply to become infinite at the stagnation point and remains infinite along the airfoil surface and its wake. On the other hand at streamlines close to the pressure side of the airfoil surface D increases monotonically as we move along a streamline. It also increases as we move closer to the airfoil until it becomes infinite at the stagnation point and the airfoil surface and its wake. The case $\psi = 0$ corresponding to the airfoil surface is not plotted because of the singular behavior of D.

It is this behavior of the drift function that produces the singular behavior of the vorticity at the airfoil surface. The regular vortical velocity, however, does not have this singular behavior.

The Vortical Velocity $\vec{u}^{(R)}$

The expression of the vortical velocity $\vec{u}^{(R)}$ is determined analytically. The analytical expression, however, depends on the drift function and the potential and stream function ϕ_0 and ψ_0 . These three quantities are explicitly known only numerically in terms of Cartesian coordinates. In order to illustrate the result of our regularization procedure we have plotted in Figures 20 and 21 the magnitudes of the streamwise and normal components of $\vec{u}^{(R)}$, versus x and ψ_0 respectively. Again, the lines stretching from left to right correspond to constant x, while those stretching in the x-direction correspond to constant ψ (along the same mean streamline). These figures clearly show that both the streamwise and normal components of $\vec{u}^{(R)}$ vanish at the airfoil surface as predicted by the theory. The normal component U_n exhibits, however, very strong gradient near the airfoil surface, while the streamwise component U_t has a rather smooth behavior.

The Potential Function ϕ

The function ϕ is a solution to the integral equation (14) which after modification became equation (19). Figures 22 and 23 show plots of the real and imaginary part of ϕ versus the airfoil arc length counted from the pressure side trailing edge. The vertical solid lines at S = 1.97 and 2.02

correspond to the locations of the stagnation points. Note that ϕ is essentially a smooth curve, with strong gradients only in the stagnation point area.

The Unsteady Pressure

The unsteady pressure is the material derivative of ϕ. Figures 24 and 25 show plots for the real and imaginary components of the unsteady pressure for two different airfoils also plotted on these figures. For a thin airfoil at zero-incidence (Fig. 24), the unsteady pressure is most important near the leading edge. This result is similar to that of the flat plate case where the pressure is infinite at the leading edge. For a 10% thickness airfoil, the real component of the pressure in-phase with the gust velocity does not exhibit large values near the leading edge, while the imaginary out-of-phase component has a large peak in this area.

It is of course not possible to draw a conclusion about the behavior of the pressure from the present results. More studies are needed to determine the essential features of the unsteady pressure in terms of the many parameters entering the problem.

The Unsteady Lift

The present theory is the most general aerodynamic analysis for an arbitrary shaped airfoil subject to three-dimensional gust. It contains all previously studied cases such as that of Sears [3], the nonlinear 2-D second order theory [8], and Graham [22] oblique gust theory for a flat plate airfoil. Therefore we started by comparing our results with the linear theory of Graham and our second order theory.

We first calculated the real and imaginary parts of the lift for a flat plate airfoil subject to an oblique gust with a spanwise wave number $k_3 = 1.0$. Figure 26 shows the imaginary part of the lift (out-of-phase component) versus the real part of the lift (in-phase component) for different values of the reduced frequency k_1. Because we cannot take a zero-thickness airfoil, we have considered a 5% thickness airfoil. Our results are plotted in a similar way to the flat plate case and indicate an excellent agreement up to reduced frequencies of order 2. Above $k_1 = 2$, a slight difference in the phase of the lift appears to exist between the linear theory and ours. This could be attributed to the 5% thickness of our airfoil. However, it could also result from a reduced accuracy in the numerical scheme as k_1 increases to larger values. A further investigation of the high frequency case is underway to clarify this point.

The second comparison we have carried out was with our previous results [24]. In [24] we have derived an analytical formula for the lift of a cambered airfoil at incidence to the mean flow and subject to a two-dimensional gust. Figure 27 shows the result of the second order theory (SOT) compared with our present (PRT) calculation for a 5% thickness airfoil. Again, we see an excellent agreement up to a reduced frequency of order 2. Figure 28 shows similar results for the same airfoil but with 5° angle of attack.

These comparisons show clearly the good accuracy of our computed results. Figure 29 shows the three-dimensional effect of the gust, where we have compared the lift resulting from a 2-D gust with that resulting from a three-dimensional gust $k_3 = 1$, the reduced frequency k_1 was varied from 0.25 to 2.5 . The gust transverse wave number is $k_2 = k_1$. It is noted that k_3 has a very significant effect on both the magnitude and the phase of the lift low frequency. In fact for k_1 between 0.25 and 0.5, both magnitude and phase of the 3-D gust are almost constant. This result is in harmony with certain results given by Graham [25] and in fact should be expected in view of the fact that the airfoils considered departs only very slightly from a flat plate airfoil.

The comparisons we have carried out with the 2-D linear Sears results, 3-D linear Graham calculations, and our previously developed 2-D nonlinear theory, are not yet complete. For example, it is very important to determine the validity of the 2-D nonlinear theory as the mean lift of the airfoil is increased. This, of course, can now be done by a more detailed comparison of our present work which encompasses all previous cases as just a particular case assigned to certain parameters. An investigation comparing all these theories is being carried out.

V. COMPARISON BETWEEN THEORY AND DATA FOR HIGHLY LOADED OSCILLATING CASCADES

We have previously developed under AFOSR sponsorship an aerodynamic theory for oscillating airfoils in cascade with arbitrary airfoil and cascade geometry. The airfoils could have translational and rotational oscillations with circumferential modes modeled as an interblade phase angle in the oscillatory motion. The theory was published in [27]. The analysis was later applied to calculate turbine blade flutter in [28] and [29]. The results clearly show that coupled bending and torsional modes could produce flutter at typical operating conditions for highly loaded blades. In an "Overview of NASA/AF/NAVY Symposium on Aeroelasticity of Turbine Engines [32], Professor Sisto noted the importance of this result and stated that "Experimental confirmation of the validity of this conclusion should be anticipated by the R&D community," [p.7].

At UTRC, Dr. Franklin Carta has been investigating the pressure distribution on oscillating cascades in pitching oscillation. Dr. Carta's research is sponsored by NASA Lewis Research Center. The above results encouraged us to cooperate for a comparison between theory and experiment. Dr. Carta visited us at Notre Dame and provided us with details regarding his experimental condition. The geometry of his airfoils and the cascade parameters are given in Table 1. We carried out a study of his cascade for interblade phase angles varying from 0 to 360°. Similar comparisons were also carried out with Verdon and Caspar [33], also from UTRC, using a purely numerical code. The results were presented in [34].

Figure [30] shows essentially this comparison for a reduced frequency, $k = 0.122$ and four values of the interblade phase angle σ. The results show excellent agreement between our analysis, Verdon and Caspar numerical code, and the experimental data. Note, however, the strong deviation of the real part of the pressure in the Verdon/Caspar results near the trailing edge. This deviation is caused by the difficulty of accurately capturing the singular behavior in the unsteady pressure at a sharp trailing edge with a finite difference scheme.

VI. FUTURE PLANS

Our past and present work under AFOSR sponsorship has always covered a broad range of topics which have evolved from the initially stated objectives. This is particularly true for the period covered by the present report.

(i) We started out to study the aerodynamics of airfoils subject to three-dimensional unsteady vortical disturbances. A problem of great interest in aerodynamics and aeroelasticity.

(ii) This led us to develop new mathematical methods which apply to large structure disturbances acting upon streamlined bodies with wakes such as airfoils and blades in turbomachines subject to rotor-stator interaction or inlet distortion.

(iii) Later we generalized the mathematical procedure to cover the equally important cases of small structure turbulence disturbances interacting with streamlined and bluff bodies. This is known as the rapid distortion theory of turbulence.

(iv) The mathematical theory led to a uniformly valid splitting theorem for unsteady vortical and entropic disturbances of potential flows.

(v) This led us to initiate an investigation to study the change in turbulance near the stagnation point of a flow. A problem of great interest in studying the stability of a flow as it interacts with a body.

The above shows the broad scope of our research interests which essentially are in the following areas:

- Unsteady aerodynamics of airfoils and cascades
- Flutter and stability analysis of airfoils, cascades and assembled systems
- Mathematical methods: analytical and numerical
- Unsteady flows

Our future plans are therefore determined by ongoing state-of-the-art developments in the aerodynamics and the stability analysis of turbomachine systems and their relevance to technological developments.

Essentially, these are the problems we shall be pursuing and developing in the near future:

1. <u>Three-Dimensional Vortical Disturbances Acting Upon Loaded Airfoils</u>

We shall complete the analytical analysis of the aerodynamics of an airfoil of arbitrary geometry subject to 3-D gust. This will include a parametric study of the unsteady forces in terms of the airfoil geometry, the mean flow incidence, and the frequency and the direction of the gust as defined by the three wave numbers. The comparison with linear 3-D theories and nonlinear 2-D theories will also be completed. This study should determine the significant parameters affecting the aerodynamics of a 3-D gust.

2. <u>Interaction of Swirling Vortices with Loaded Airfoils</u>

In turbomachines, major upstream disturbances are produced either by viscous wake defects or by swirling vortices usually emanating from upstage tips. It is important for the designer to know which effect produces higher fluctuating lift and whether a design shape optimization can reduce the unwanted fluctuations in the forces. This study can be carried out as an application to our general theory of a 3-D gust.

3. High Frequency 3-D Disturbances Acting Upon Loaded Airfoils at High Subsonic Mach Numbers

This is the case closest to operating conditions for high-speed machines. The treatement of the general problem requires significant mathematical and numerical efforts. However, if one assumes the reduced frequency to be high, a simplification of the mathematical equation governing the problem can be brought about and leads to a formulation similar to the one we have developed for a low-Mach number.

4. 3-D Vortical Disturbances Acting Upon Loaded Cascades

Depending on the solidity, the cascade effects can be very important. Even for a small solidity, resonance effects can take place in a cascade. The extension of our theory to a cascade of airfoils can be done with no difficulty except for the added computational time due to define the mean flow of the cascade. The incorporation of high Mach number and high frequency to the cascade analysis will be a very important contribution to a more accurate evaluation of flutter and noise generation in turbomachines.

5. Change in Turbulence and Pressure Fluctuation Near a Stagnation Point

As a spin-off of our splitting theorem, we can study the change in the characteristics of a turbulent flow near the stagnation point of an airfoil or a bluff body. This is important to determine the stability of a flow as it interacts with the body. Separation and boundary-layer stability are affected by the level of turbulence and the pressure fluctuation near a stagnation point.

6. Aeroelastic Characteristcs of Loaded Blade Disc Assemblies

All present aeroelastic stability studies of blade disc assemblies use simplified aerodynamic codes developed from linear unsteady aerodynamic theories wherein the blades are approximated by flat plates. Our previously developed theory for loaded cascades oscillating with circumferential modes showed significant changes in the stability boundary from the results of flat plate cascades. We feel that a study is needed to incorporate a more realistic aeroelastic model with our aerodynamic theory. Such a study will define a more accurate flutter boundary for loaded blade disc assemblies.

VII. REFERENCES

1. Kemp, N.H., and Sears, W.R., "Aerodynamic Interference Between Moving Blade Rows", Journal of Aeronautical Sciences, Vol. 20, No. 9, September 1953, pp. 585-598.

2. Atassi, H., "Three-Dimensional Periodic Disturbances Acting Upoon Airfoils in Cascade", Aeroelasticity in Turbomachines, Ed. P. Suter, Juris-Verlag, Zurich, pp. 383-398, 1981.

3. Sears, W.R., "Some Aspects of Nonstationary Airfoil Theory and Its Practical Applications", Journal of Aeronautical Sciences, Vol. 8, No. 3, 1941, pp. 104-198.

4. Graham, J.M.R., "Similarity Rules for Thin Airfoils in Nonstationary Flows," J. Fluid Mech., Vol. 43, pp. 753-766, 1970.

5. Lane, F., and Friedmann, M., "Theoretical Investigation of Subsonic Oscillatory Blade-Row Aerodynamics", NACA TN 4136, February 1958.

6. Whitehead, D.S., "Force and Moment Coefficients for Vibrating Airfoils in Cascade", British A.R.C., R&M, No. 3254, February 1960.

7. Atassi, H. and Hamad, G., "Sound Generated in a Cascade by Three-Dimensional Disturbances Convected in a Subsonic Flow", AIAA Paper No. 81-2046, 7th Aerodynamic Conference, Palo Alto, California, October 5-7,1981.

8. Goldstein, M.E., and Atassi, H., "A Complete Second Order Theory for the Unsteady Flow About an Airfoil Due to a Periodic Gust", J. Fluid Mech., Vol. 74, 741-765, 1976.

9. Hunt, J.C.R., "A Theory of Turbulent Flow Round Two-Dimensional Bluff Bodies," J. Fluid Mechanics, 61, 625-706, 1973.

10. Batchelor, G.K., and Proudman, I., "The Effect of Rapid Distortion of a Fluid in Turbulent Motion," Quant. J. Mech. Appl. Math., 1, 83, 1954.

11. Ribner, H.S. and Tucker, M., "Spectrum of Trubulence in a Contracting Stream," NACA Report No. 1113.

12. Batchelor, G.K., An Introduction to Fluid Dynamics, Cambridge University Press, (p. 276), 1967.

13. Goldstein, M.E., "Unsteady Vortical and Entropic Distortions of Potential Flows Round Arbitrary Obstacles", J. Fluid Mechanics, Vol. 89, Part 3, pp. 433-468, 1978.

14. Goldstein, M.E., and Durbin, P.A., "The Effects of Finite Turbulence by a Contracting Stream," J. Fluid Mechanics, Vol. 98, Part 3, pp. 473-508, (1980).

15. Atassi, H., "Regularization of Goldstein's Splitting of Unsteady Vortical and Entropic Distortions of Potential Flows," Invited Paper, p.354, 19th Annual Meeting -Society of Engineering Science, October 27-29, 1982, Rolla, Missouri.

16. Atassi, H., "A Uniformly Valid Splitting of Unsteady Vortical and Entropic Disturbances of Potential Flows," in preparation.

17. Morse, P.M., and Feshbach, H., Methods of Theoretical Physics, p. 138, McGraw-Hill Book Company, 1953.

18. Possio, C., "A zione Aerodinamica sul Profilo Oscillante in un Fluido Compressible a Velocita Iposanora", L'Aerotecnica, t. XVIII, fasc, 4, April 1938.

19. Whitehead, D.S., "Vibration and Sound Generation in a Cascade of Flat Plates in Subsonic Flows", A.R.C., R&M 3685, 1972.

20. Filotas, L.T., "Vortex Induced Helicopter Blade Loads and Noise", Journ. Sound and Vibration, Vol. 27, April 1973, pp. 387-398.

21. Mugridge, B.D., "Gust Loading on a Thin Airfoil", Aeronautical Quarterly, Vol. XXII, p. 301, August 1971.

22. Graham, J.M.R., Lifting Surface Theory for the Problem of an Arbitrarily Yawed Sinusoidal Gust Incident on a Thin Airfoil in Incompressible Flow", Aeronautical Quarterly, Vol. XXI, part 2, p. 182, May 1972.

23. Atassi, H., "Influence of Loading on the Sound Field of Turbomachine Blades at Low and Moderate Mach Numbers", Transportation Noise, Ed. G. Banarium and P. Dickinson, University of Utah, November 1975.

24. Atassi, H., "Effect of Loading and Rotor Wake Characteristics on the Acoustic Response of Stator Blades", AIAA Paper, 76-566, Palo Alto, California, 1976.

25. Atassi, H. and Akai, T.J., "Effect of Blade Loading and Thickness and the Aerodynamics of Oscillating Cascades", AIAA-Paper 78-227, January 1978.

26. Atassi, H., and Akai, T.J., "Aerodynamic Force and Moment on Oscillating Airfoils in Cascade", ASME Paper 78-GT-181, April 1978.

27. Atassi, H., and Akai, T.J., "Aerodynamic and Aeroelastic Characteristics of Oscillating Loaded Cascades at Low Mach Number. Part I: Pressure Distribution, Forces, and Moments", ASME Paper No. 79-GT-111. Also J. of Engineering for Power, Vol. 102, pp. 344-351, April 1980.

28. Akai, T.J., and Atassi, H., "Aerodynamic and Aeroelastic Characteristics of Oscillating Loaded Cascades at Low Mach Number. Part II: Stability and Flutter Boundaries", ASME Paper No. 79-GT-112. Also J. of Engineering for Power, Vol. 102, pp. 352-356, April 1980.

29. Atassi, H. and Akai, T.J., "Stability and Flutter Analysis of Turbine Blades at Low Speed," Aeroelasticity in Turbomachines, Ed. P. Suter, Juris-Verlag, Zurich, pp. 187-201, 1981.

30. Atassi, H. and Grzedzinski, J., "Three-Dimensional Periodic Vortical Disturbances Acting Upon an Airfoil," in preparation.

31. Lighthill, M.J., "Drift", J. Fluid Mech., Vol. 1, pp. 31-53, 1956.

32. Sisto, F., "Overview NASA/AF/NAVY Symposium on Aeroelasticity of Turbine Engines", ME-RT-81003, Steven Institute of Technology, March 1981.

33. Verdon, J.M., and Caspar, J.R., "Development of a Linear Unsteady Aerodynamic Analysis for Finite Deflection Subsonic Cascades", AIAA Paper 81-1290, 14th Fluid and Plasma Dynamics Conference, June 23-25, 1981, Palo Alto, California.

34. Carta, F.O.,"Unsteady Gapwise Periodicuty of Oscillating Cascaded Airfoils", ASME Paper No. 82-GT-163, 27th International Gas Turbine Conference, April 18-22, 1982, London, England.

Angle of Attack	α = 0.0785398 ≅ 4.5°
THICKNESS =	0.0601266 Chord
CAMBER =	0.0218513 Chord
GAP/CHORD =	0.7500880
STAGGER =	0.959930
REDUCED FREQUENCY =	0.122000

σ = INTERBLADE PHASE ANGLE

TABLE 1. United Technologies Cascade Parameters

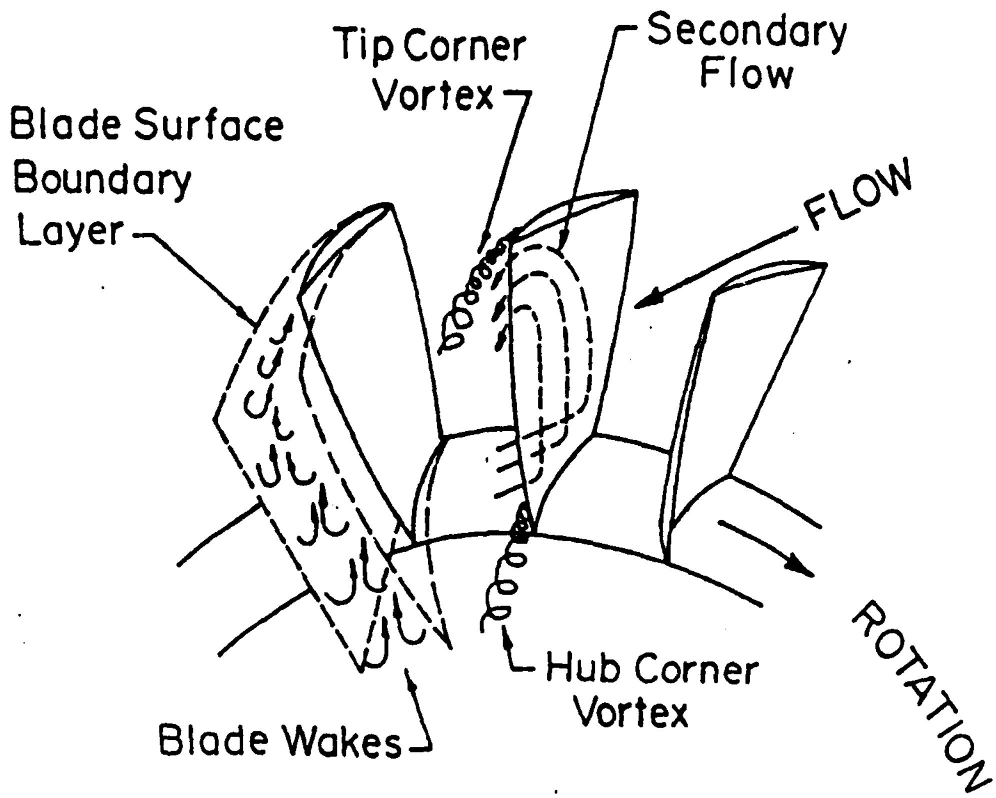


FIGURE 1. SCHEMATIC REPRESENTATION OF FLOW BEHIND A ROTOR BLADE ROW

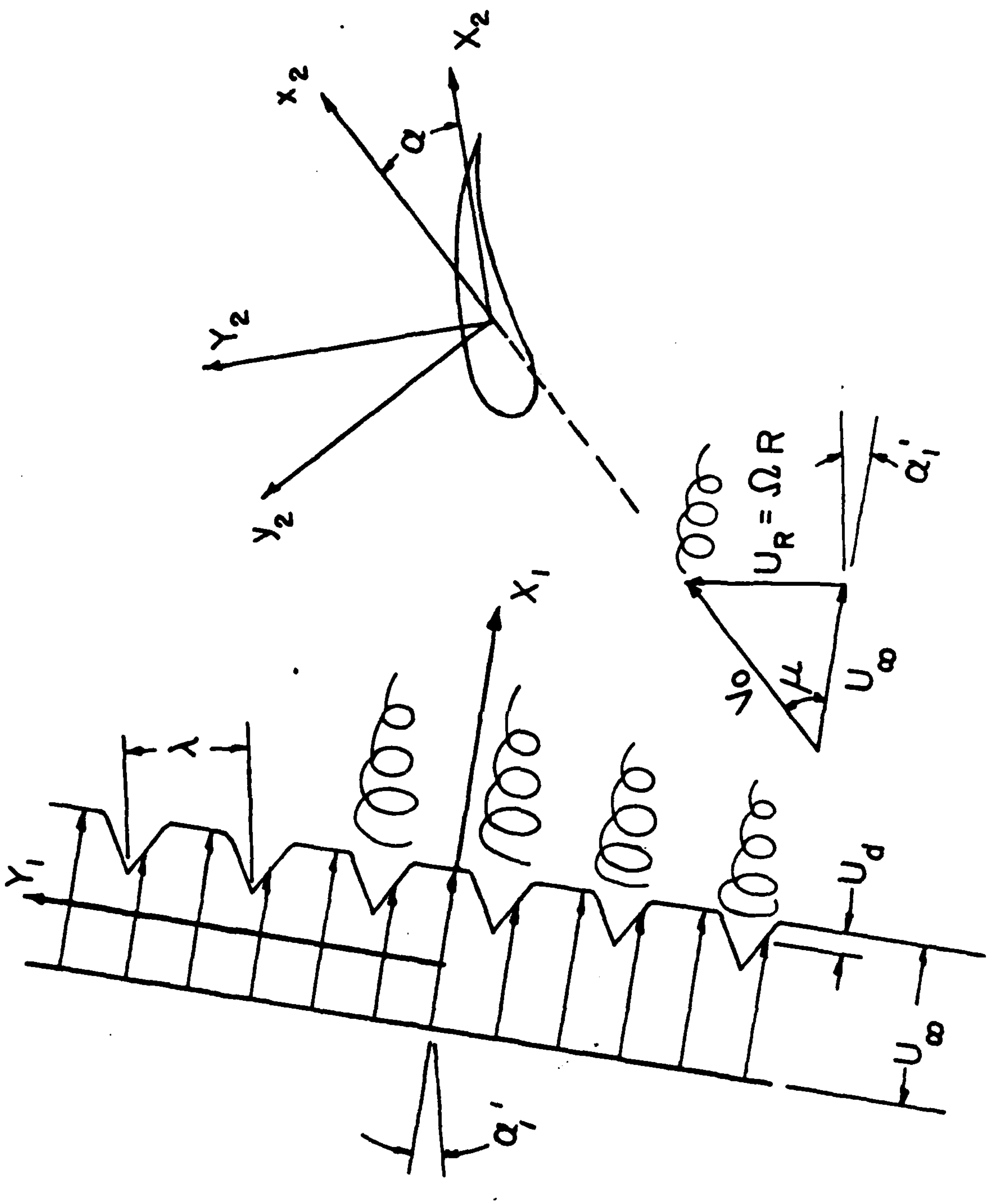


FIGURE 2. STATOR BLADE IN THE WAKE OF A ROTOR

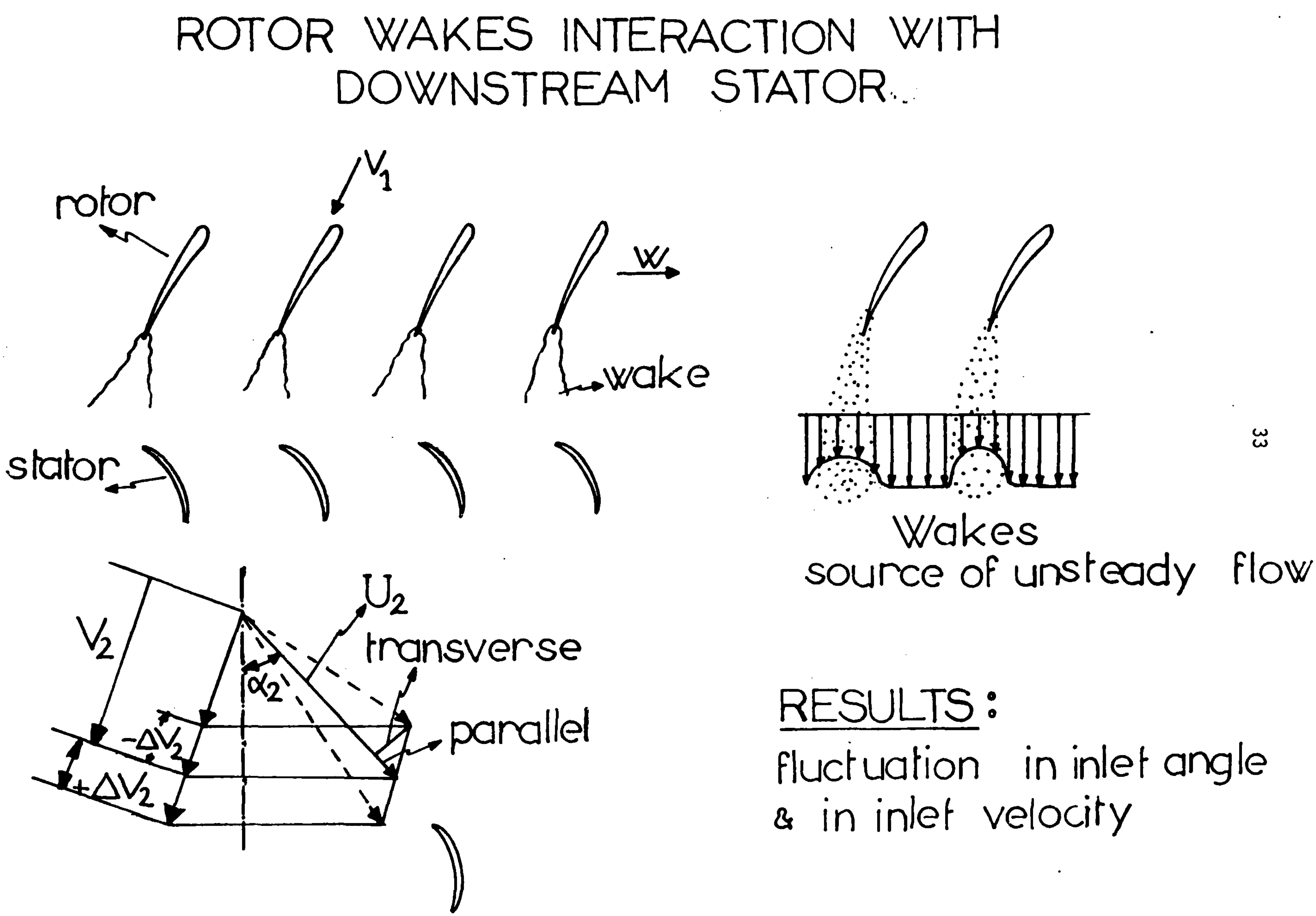


FIGURE 3. ROTOR WAKES INTERACTION WITH DOWNSTREAM STATOR

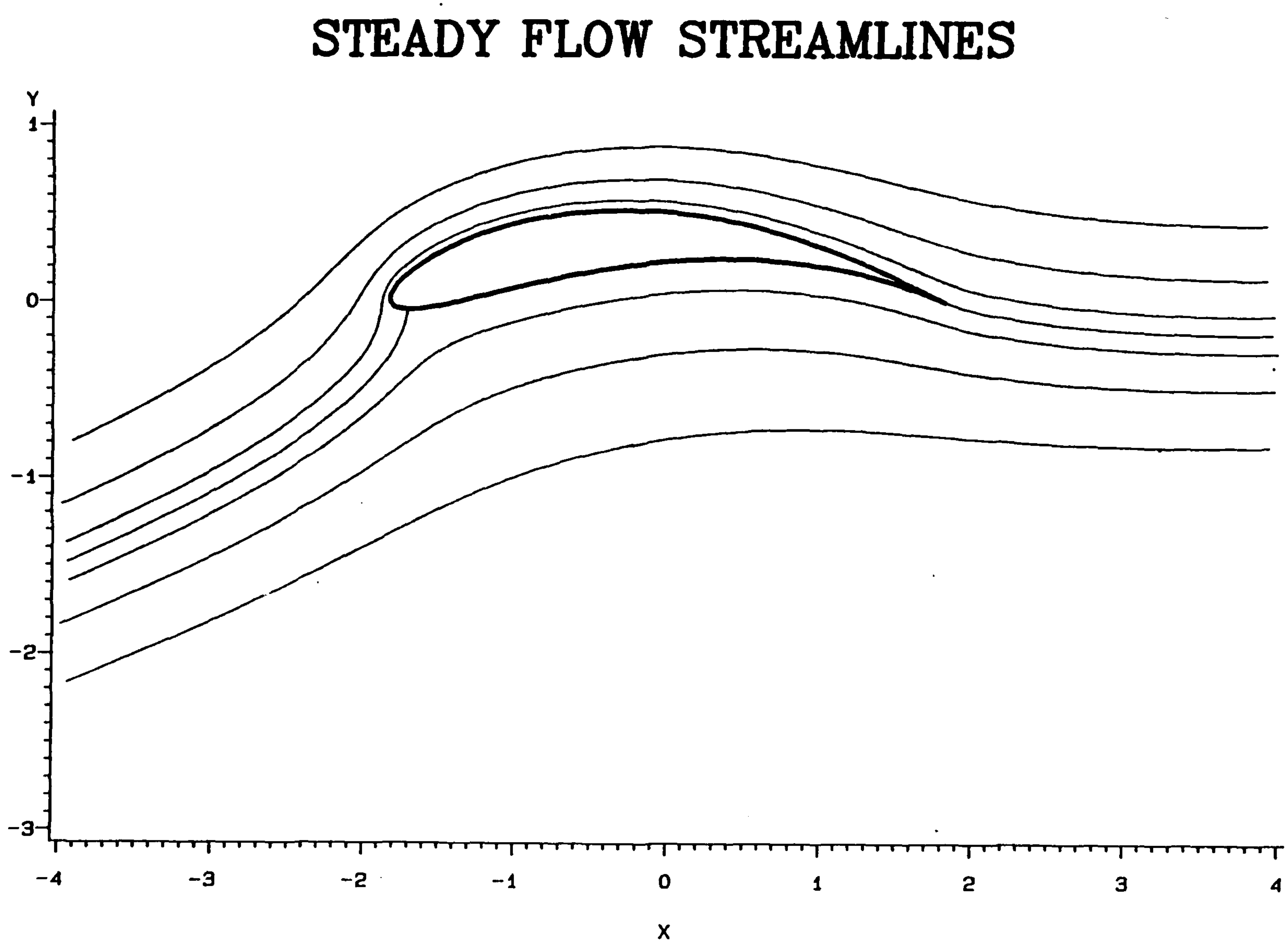


FIGURE 4. **JOUKOWSKI AIRFOIL CAMBER=0.1 THICKNESS=0.1 INCIDENCE=10 DEG**

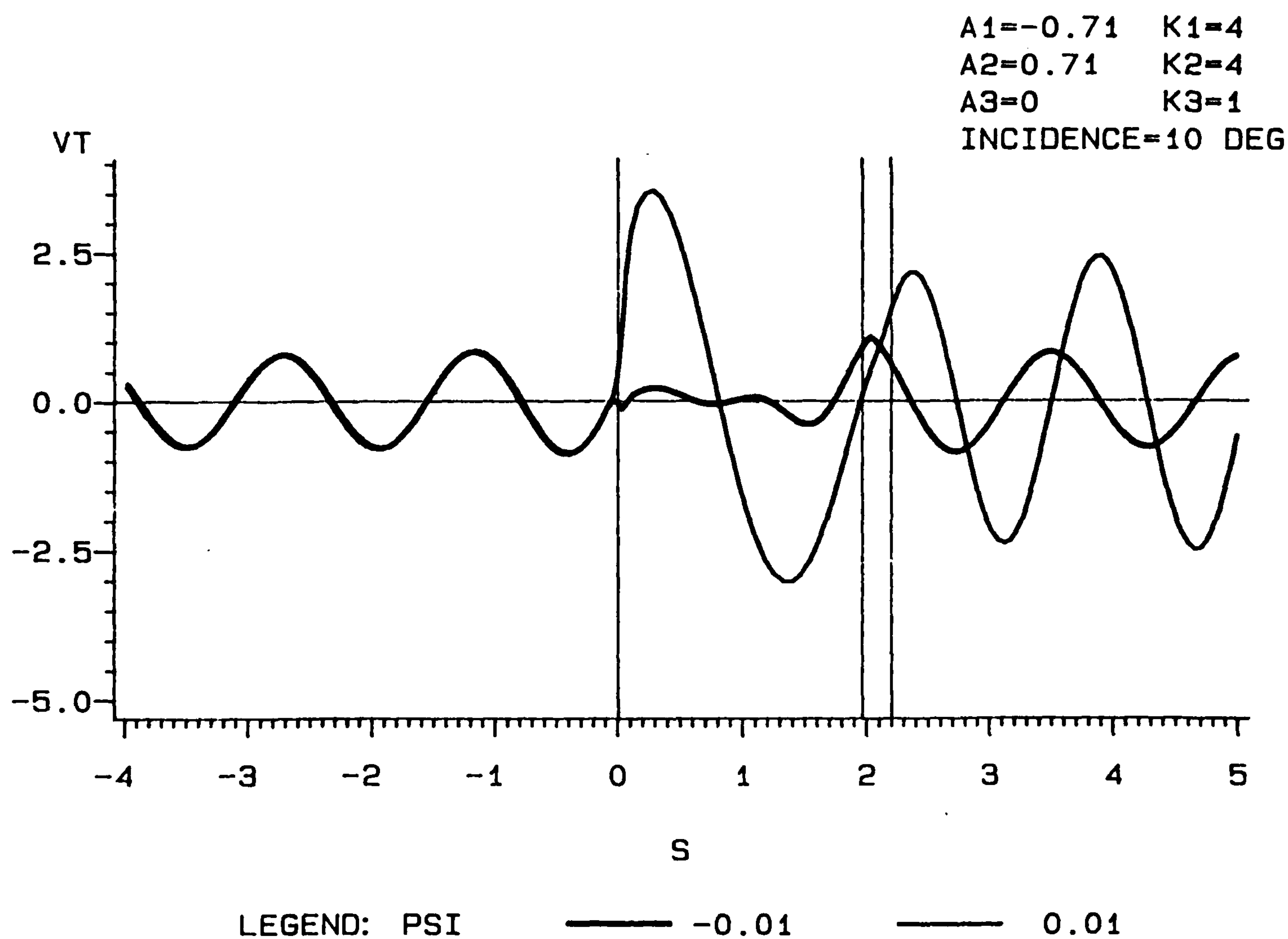


FIGURE 5. THE STREAMWISE COMPONENT OF THE VORTICITY (REAL)
JOUKOWSKI AIRFOIL: CAMBER=0.1 THICKNESS=0.1

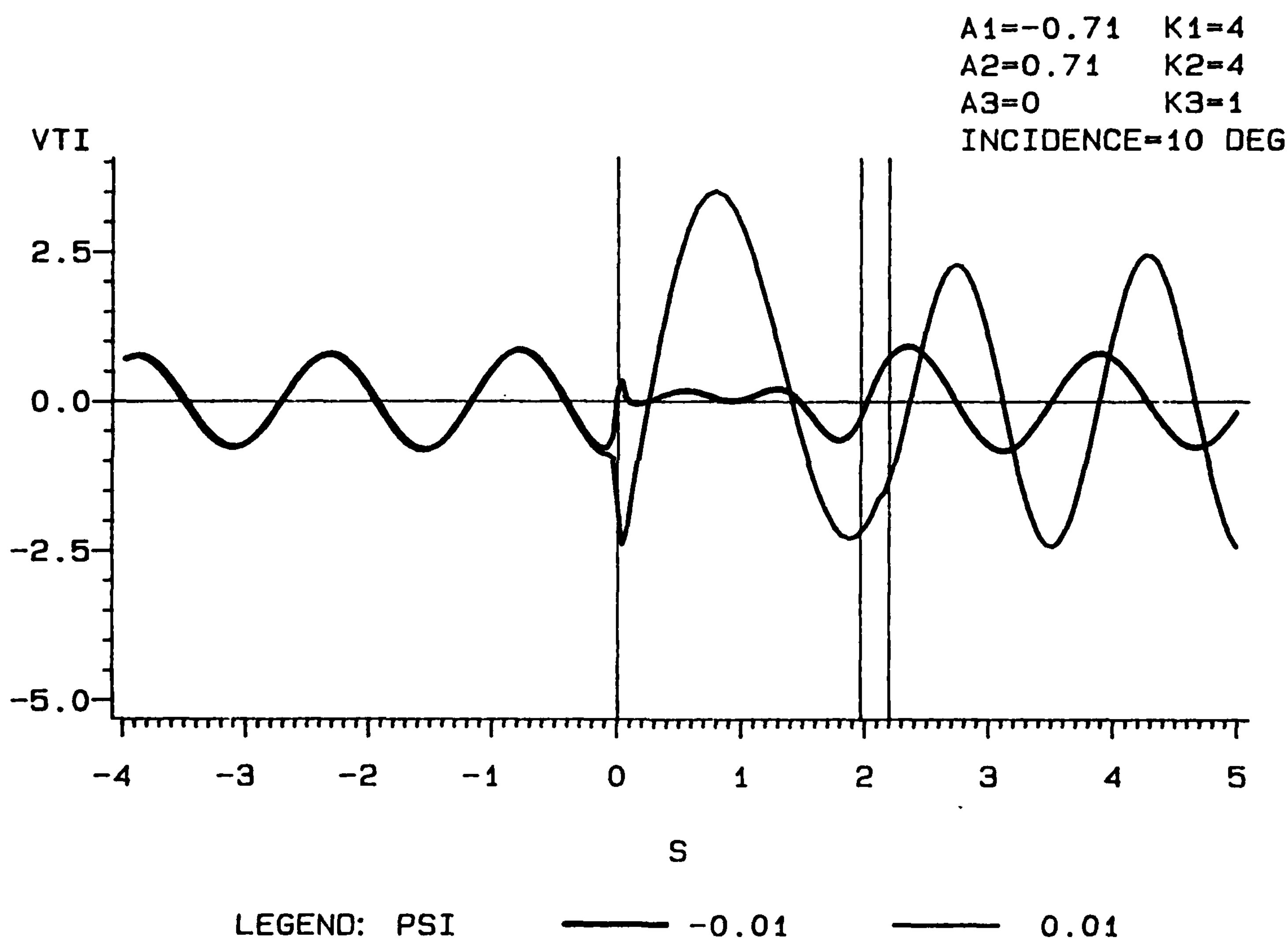


FIGURE 6. THE STREAMWISE COMPONENT OF THE VORTICITY (IMAGINARY)
JOUKOWSKI AIRFOIL: CAMBER=0.1 THICKNESS=0.1

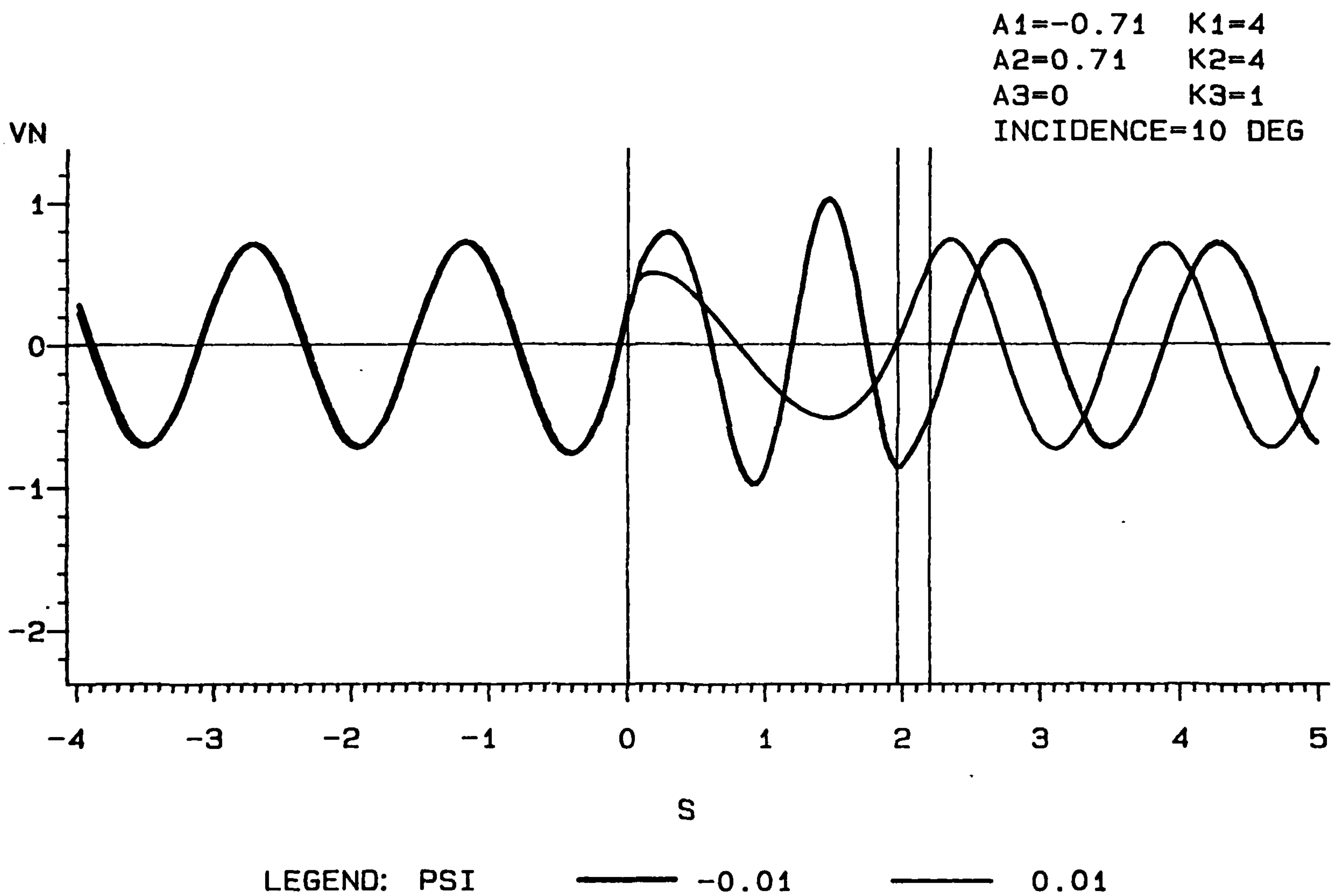


FIGURE 7. THE NORMAL COMPONENT OF THE VORTICITY (REAL)
JOUKOWSKI AIRFOIL: CAMBER=0.1 THICKNESS=0.1

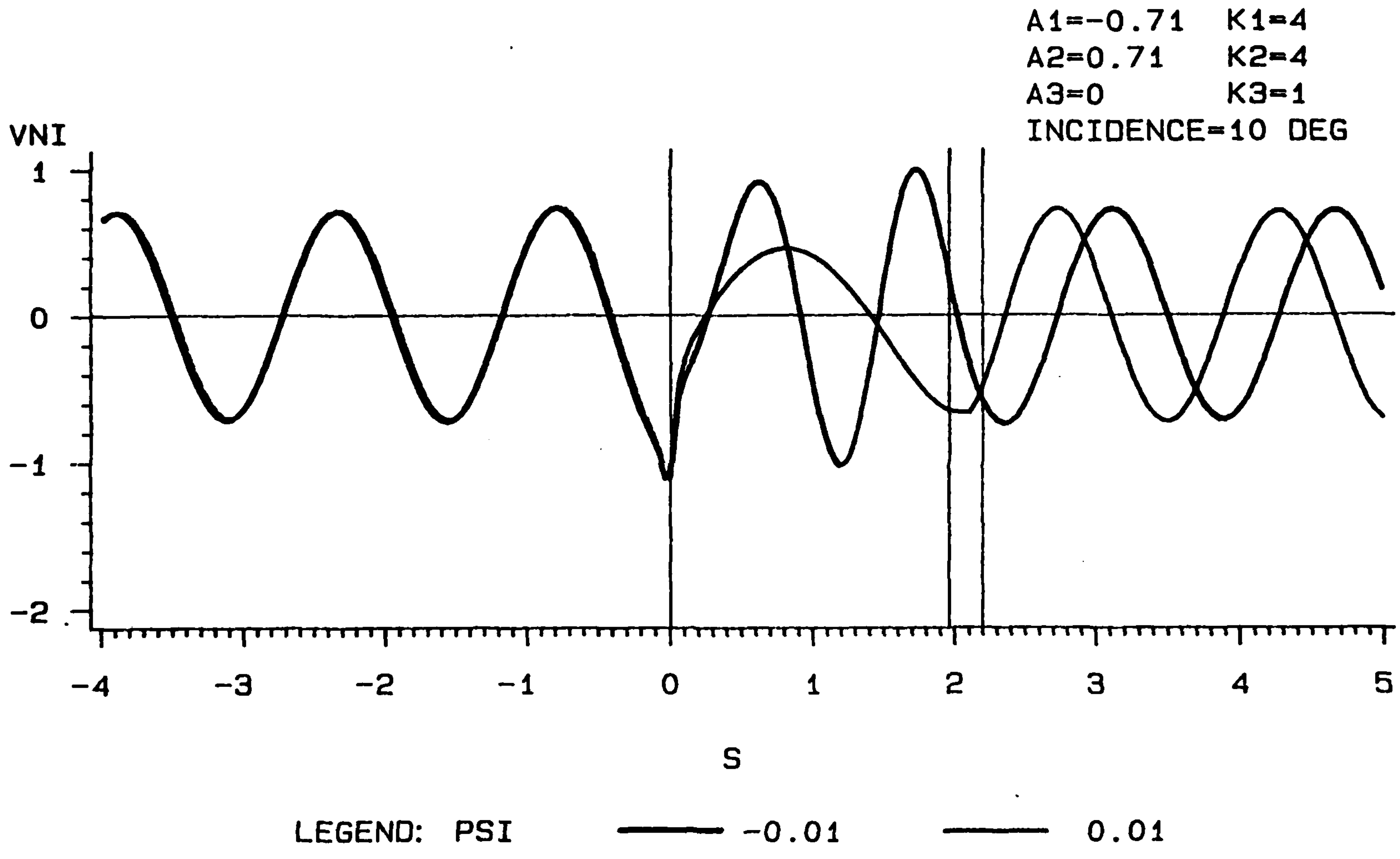


FIGURE 8. THE NORMAL COMPONENT OF THE VORTICITY (IMAGINARY)
JOUKOWSKI AIRFOIL: CAMBER=0.1 THICKNESS=0.1

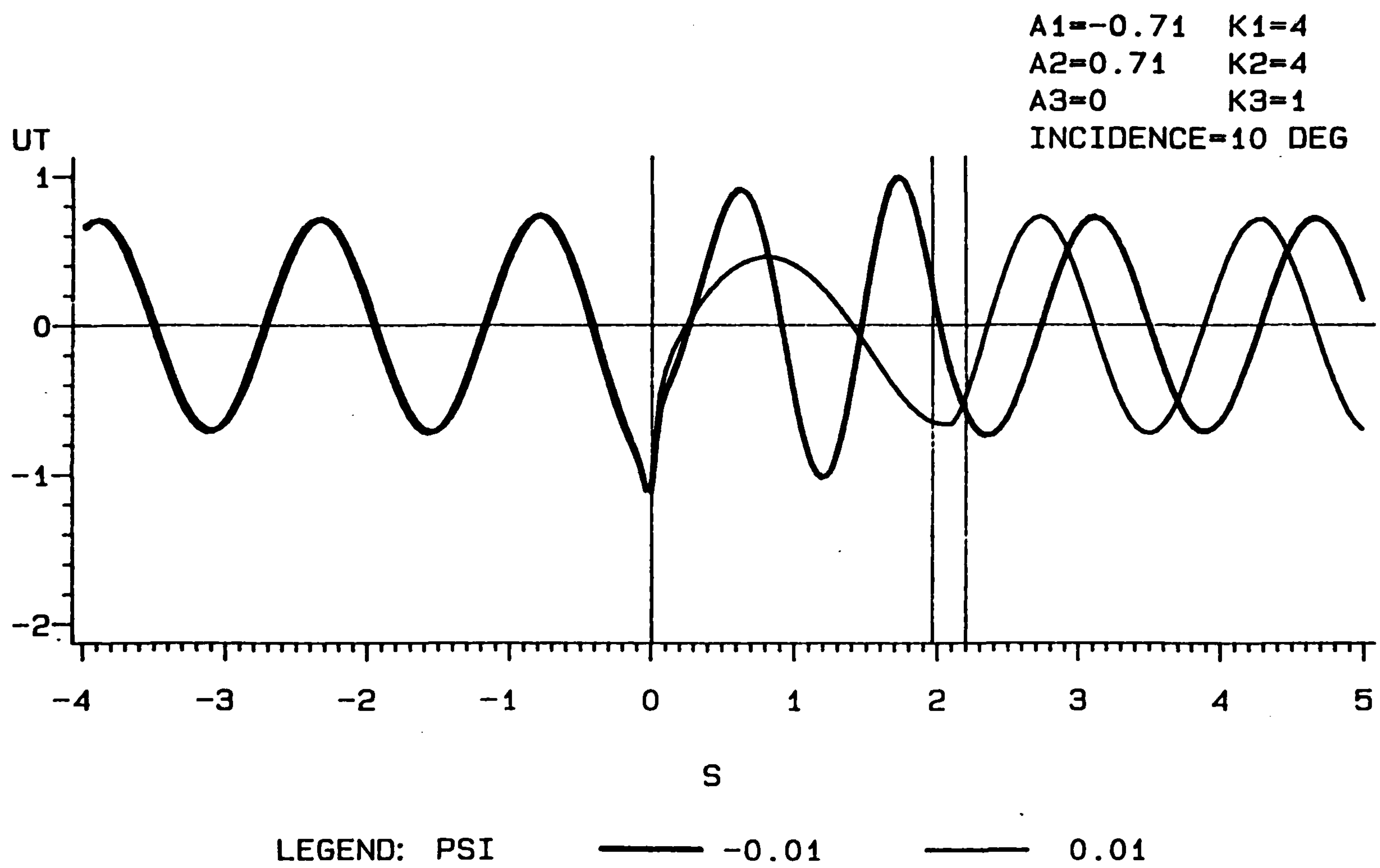


FIGURE 9. THE STREAMWISE COMPONENT OF THE VORTICAL VELOCITY (REAL)
JOUKOWSKI AIRFOIL: CAMBER=0.1 THICKNESS=0.1

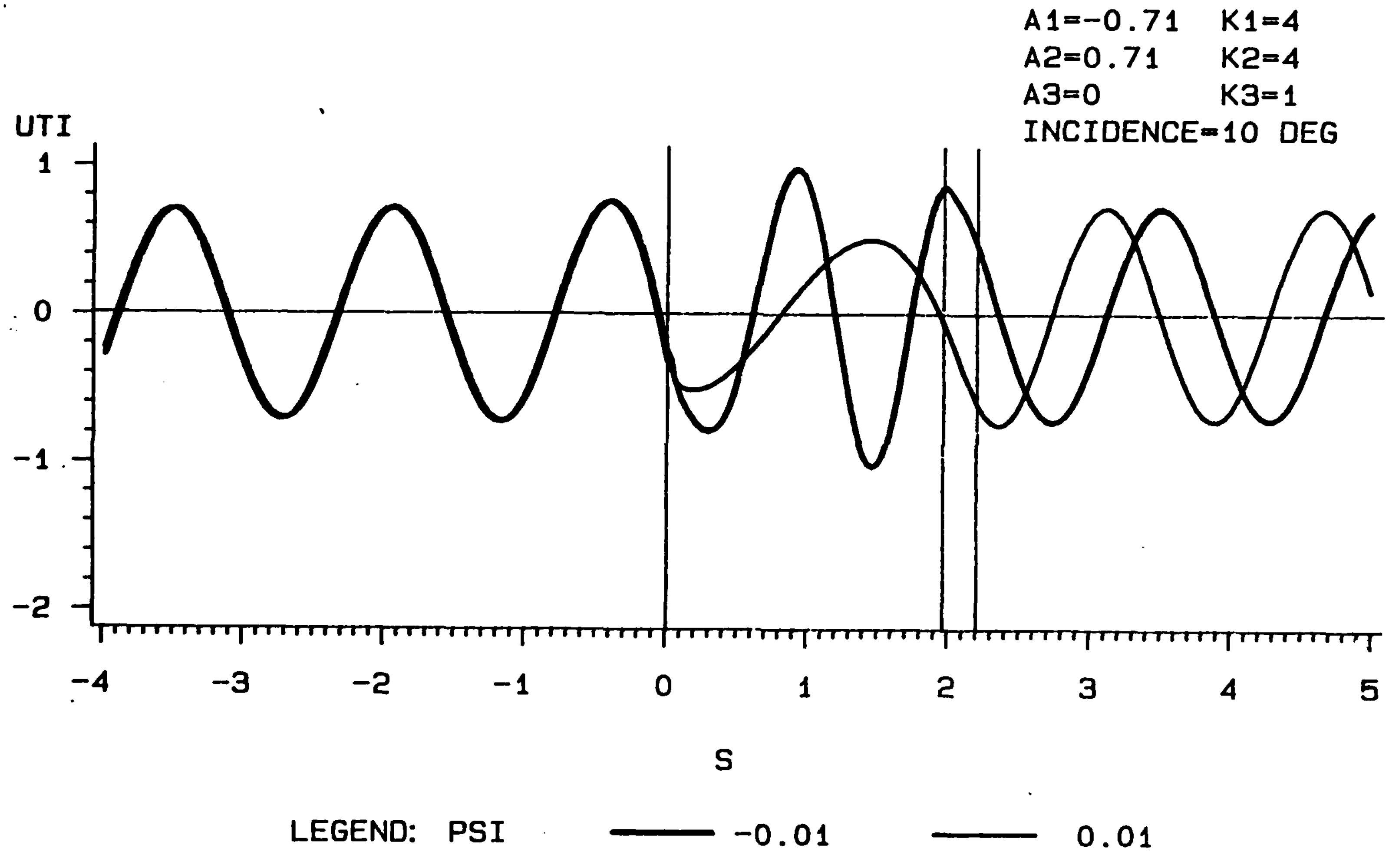


FIGURE 10. THE STREAMWISE COMPONENT OF THE VORTICAL VELOCITY (IMAGINARY)
JOUKOWSKI AIRFOIL: CAMBER=0.1 THICKNESS=0.1

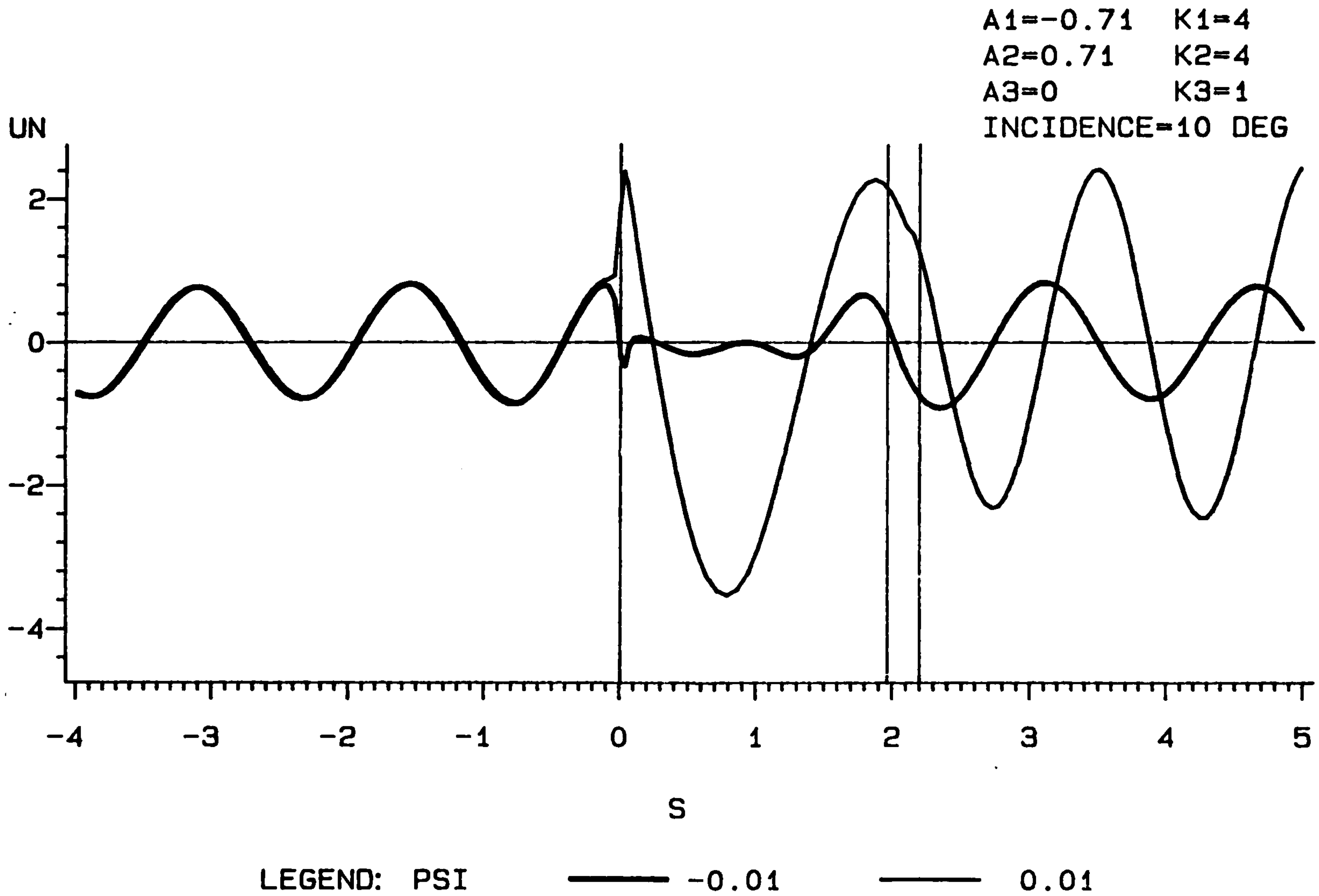


FIGURE 11. THE NORMAL COMPONENT OF THE VORTICAL VELOCITY (REAL)
JOUKOWSKI AIRFOIL: CAMBER=0.1 THICKNESS=0.1

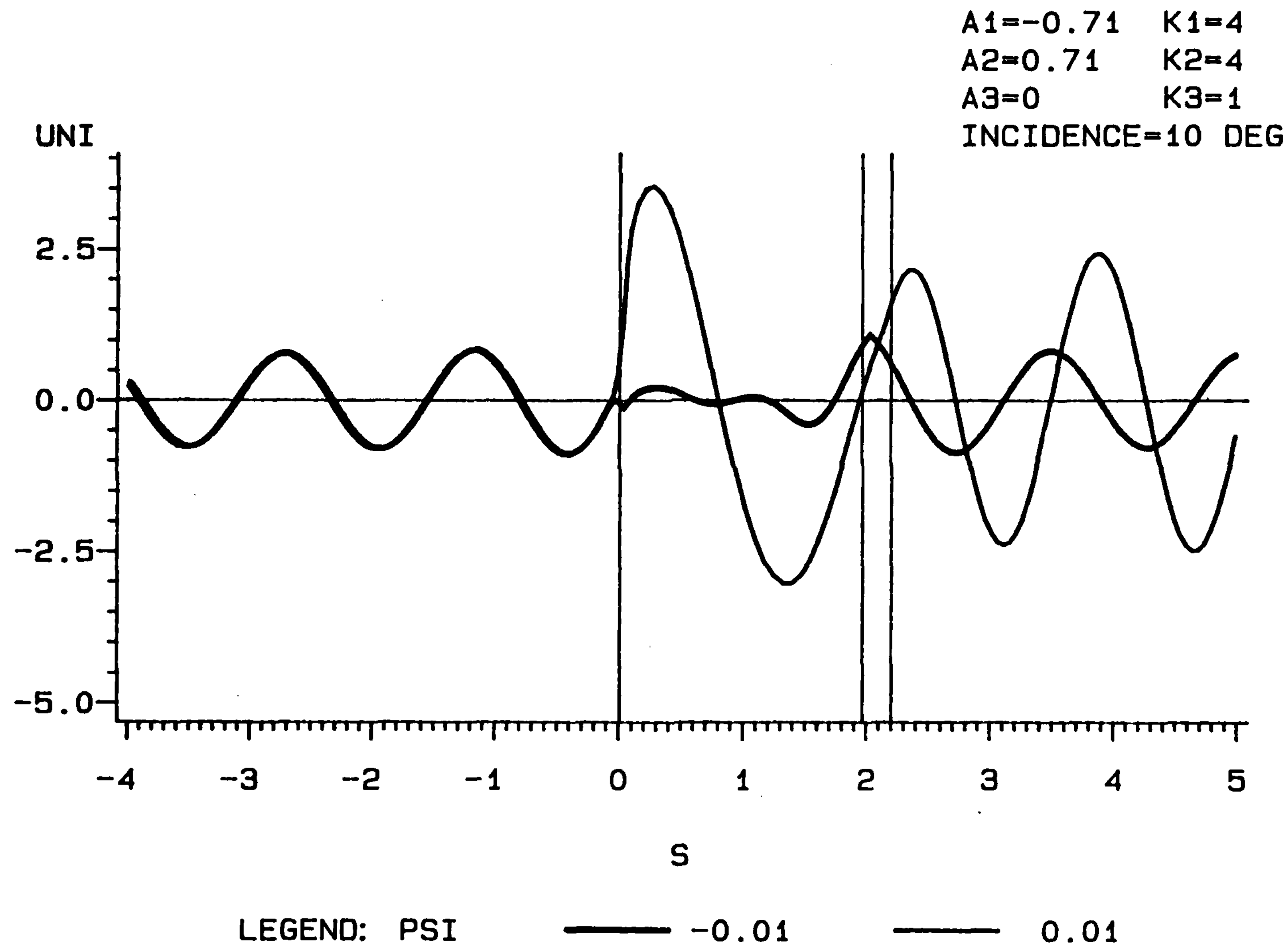


FIGURE 12. THE NORMAL COMPONENT OF THE VORTICAL VELOCITY (IMAGINARY)
JOUKOWSKI AIRFOIL: CAMBER=0.1 THICKNESS=0.1

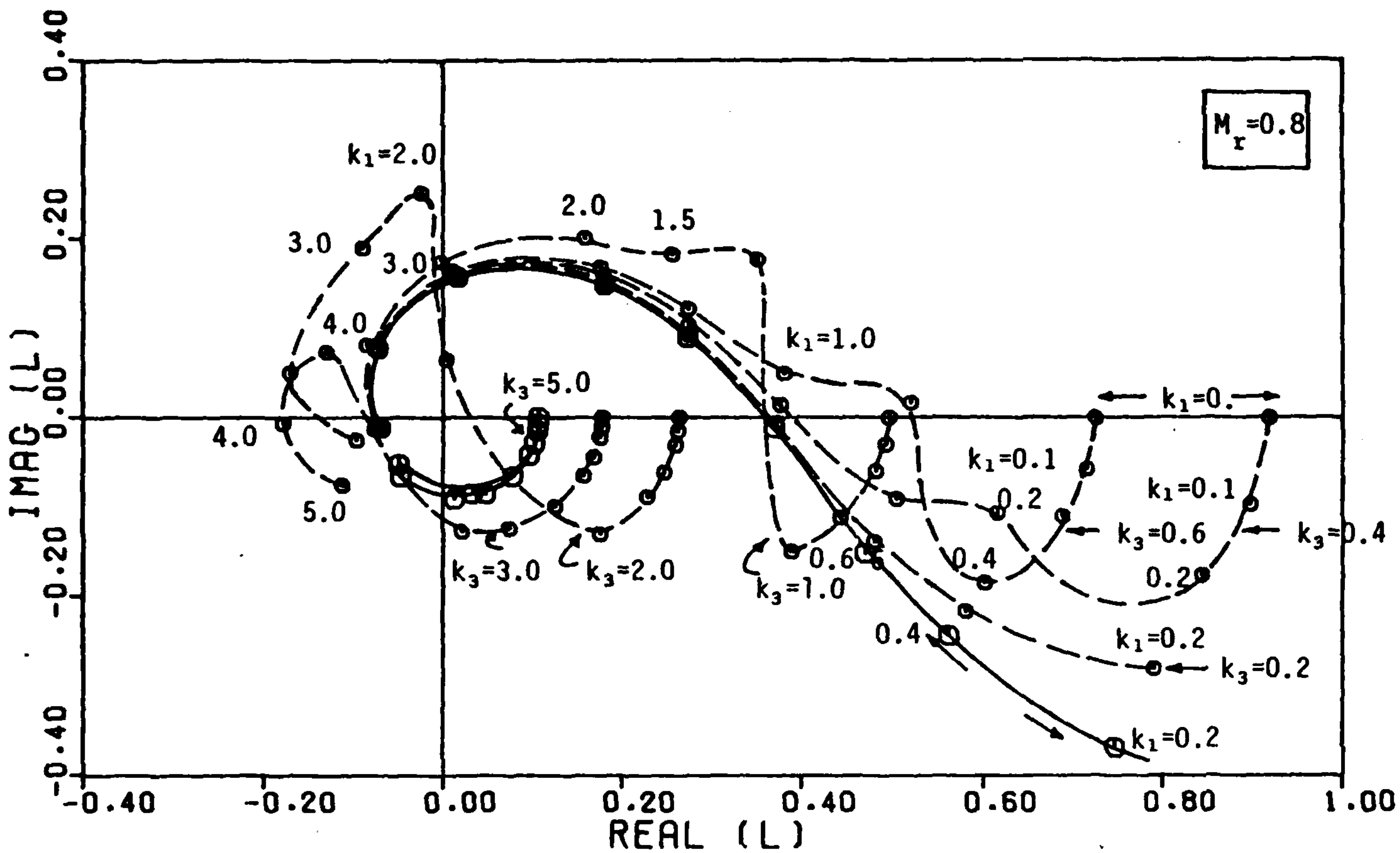


FIGURE 13. REAL AND IMAGINARY PARTS OF THE LIFT COEFFICIENT L FOR AN OBLIQUE GUST ACTING ON A FLAT PLATE FOR DIFFERENT SPANWISE WAVE NUMBER k_3. THE REDUCED FREQUENCY k_1 IS VARIED FROM 0 to 5.

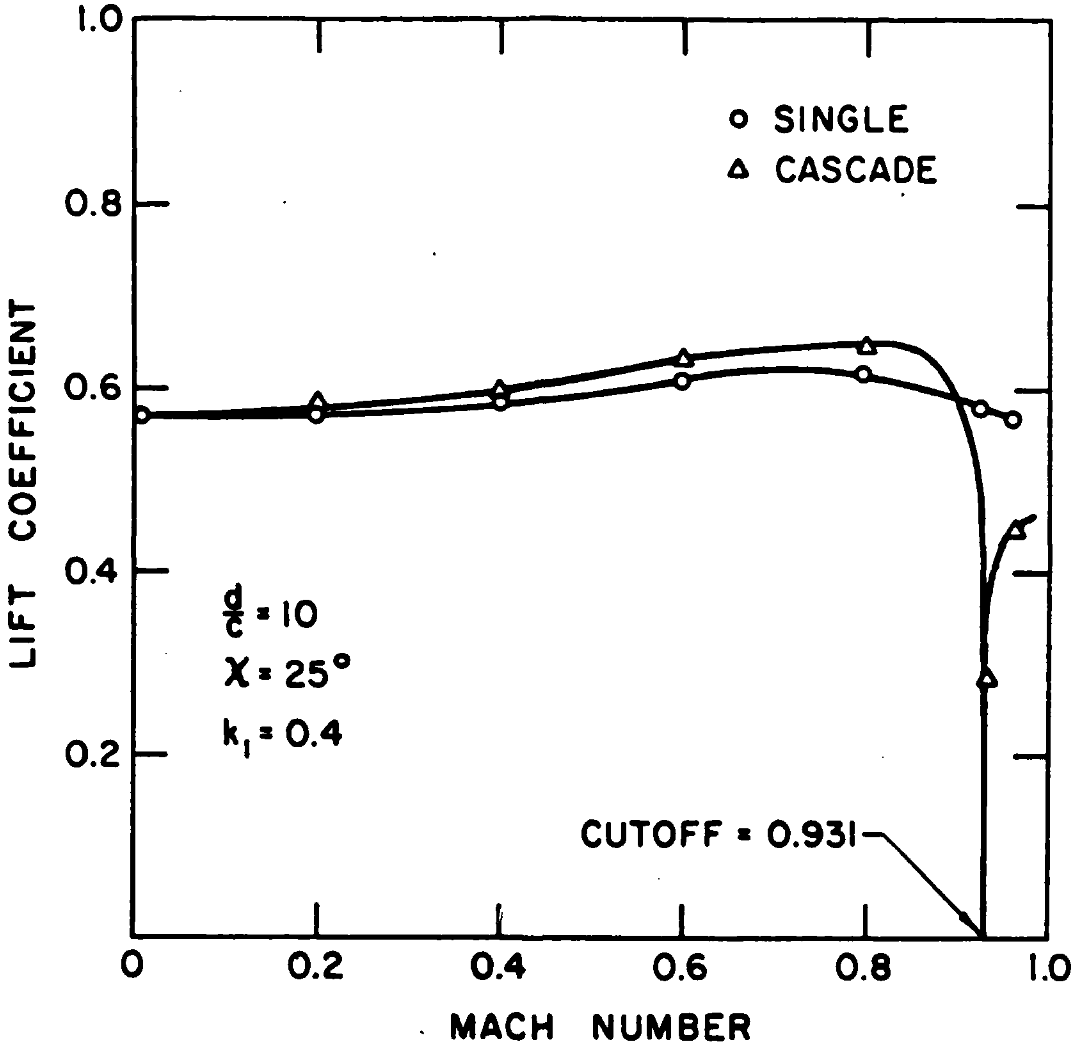


FIGURE 14. COMPARISON BETWEEN THE LIFT OF A SINGLE AIRFOIL AND A CASCADE OF VERY LARGE SPACING. RESONANCE OCCURS AT M = 0.931.

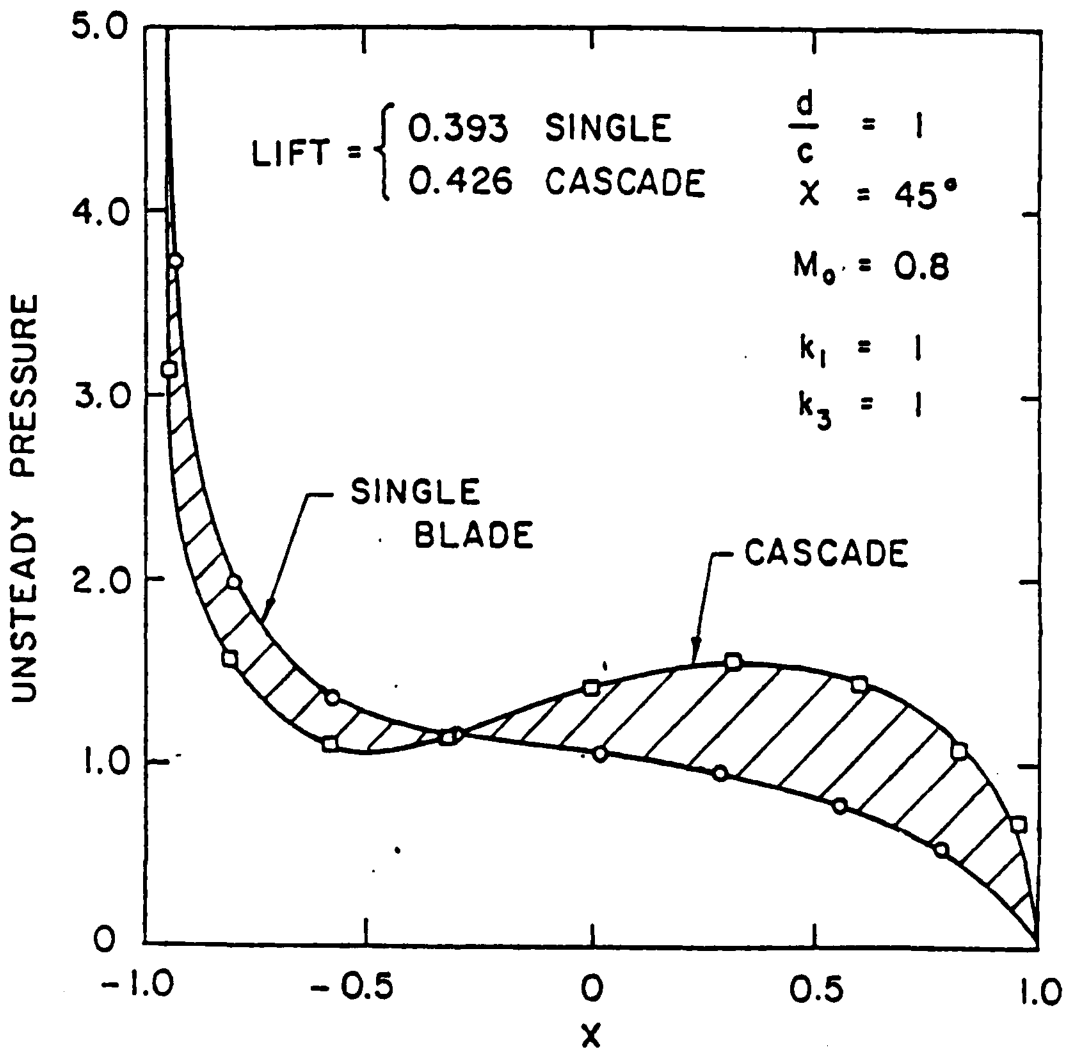


FIGURE 15. COMPARISON BETWEEN THE AERODYNAMIC PRESSURE DISTRIBUTION ALONG A SINGLE AIRFOIL AND A CASCADE BLADE. THE INTERBLADE PHASE ANGLE = 2.

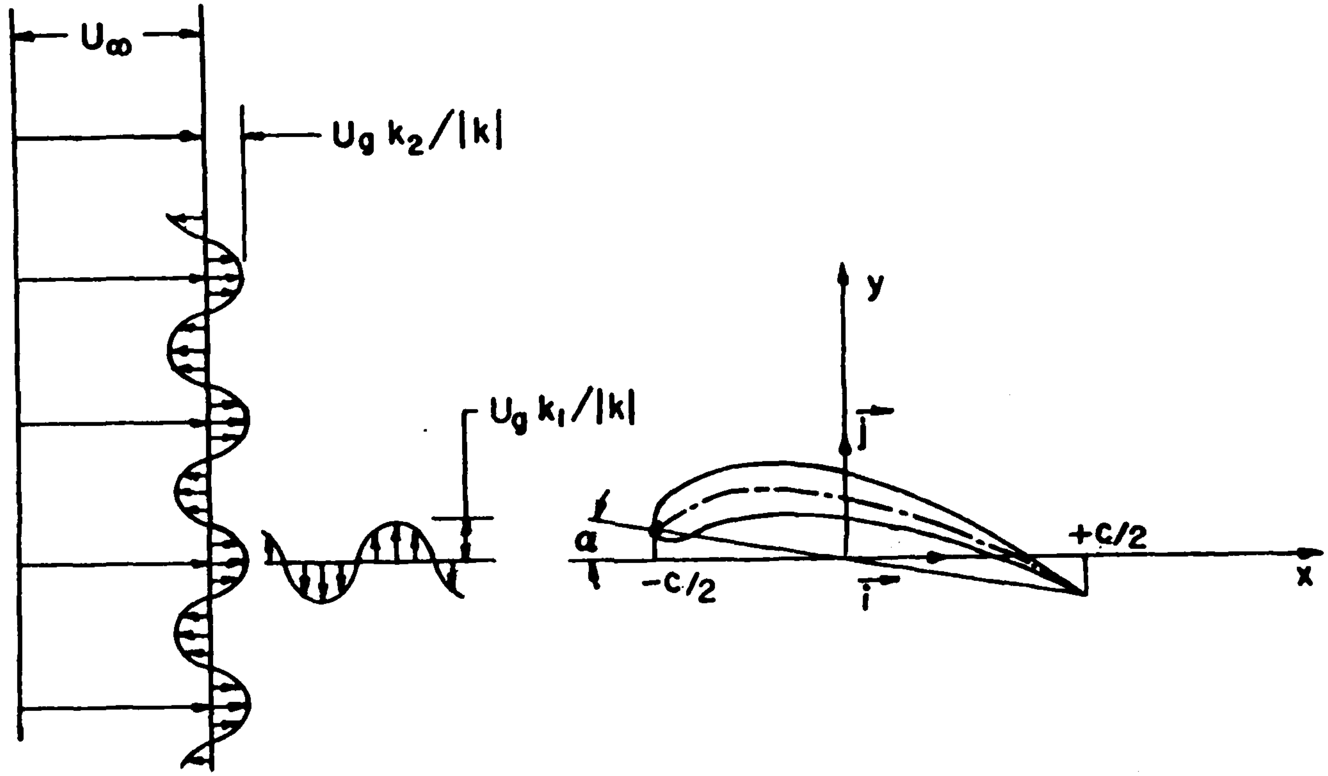


FIGURE 16. AIRFOIL IN A TRANSVERSE AND LONGITUDINAL GUST.

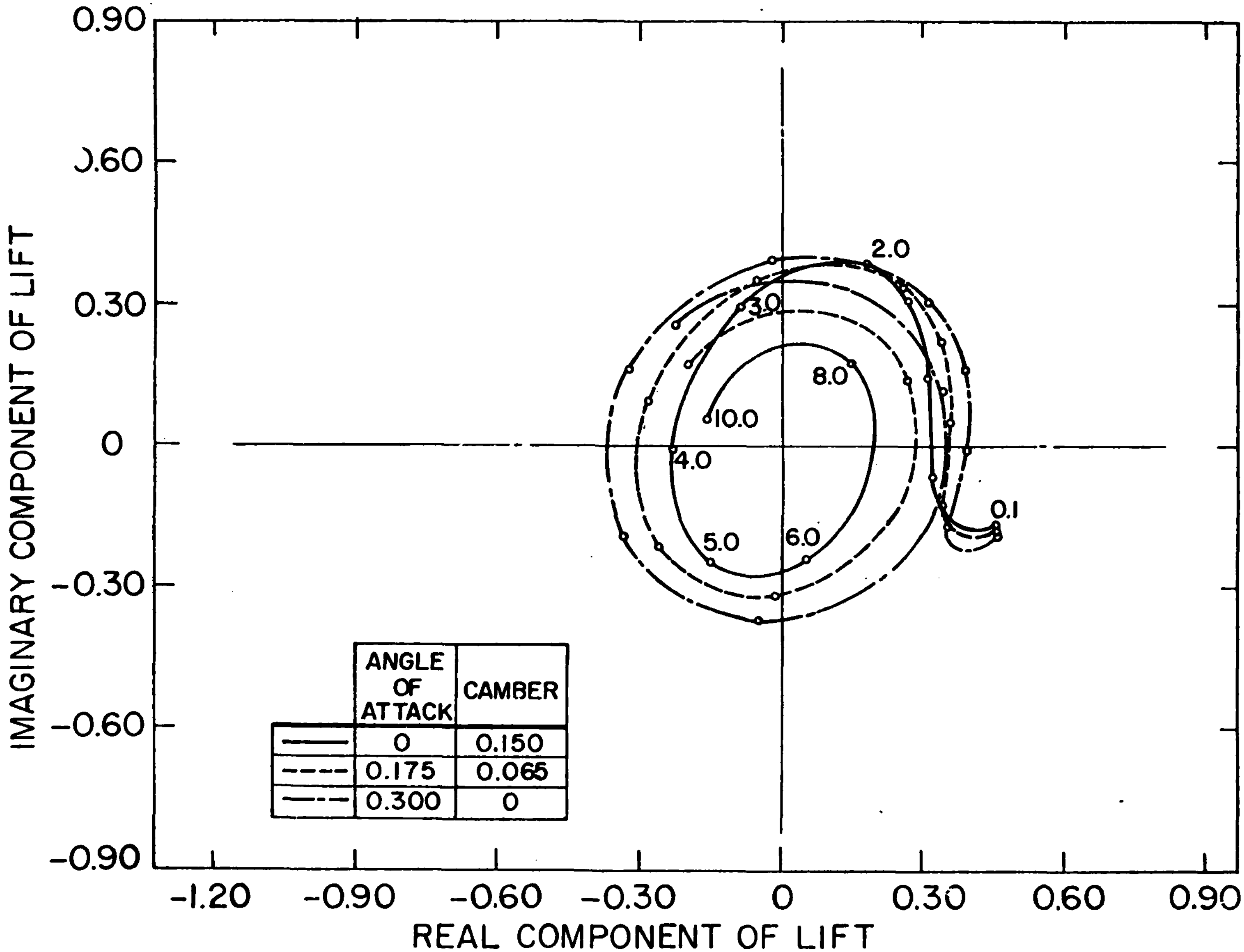


FIGURE 17. REAL AND IMAGINARY COMPONENT OF THE UNSTEADY LIFT FOR THREE AIRFOILS HAVING THE SAME MEAN LIFT. THE REDUCED FREQUENCY k_1 IS VARIED FROM 0 TO 10.

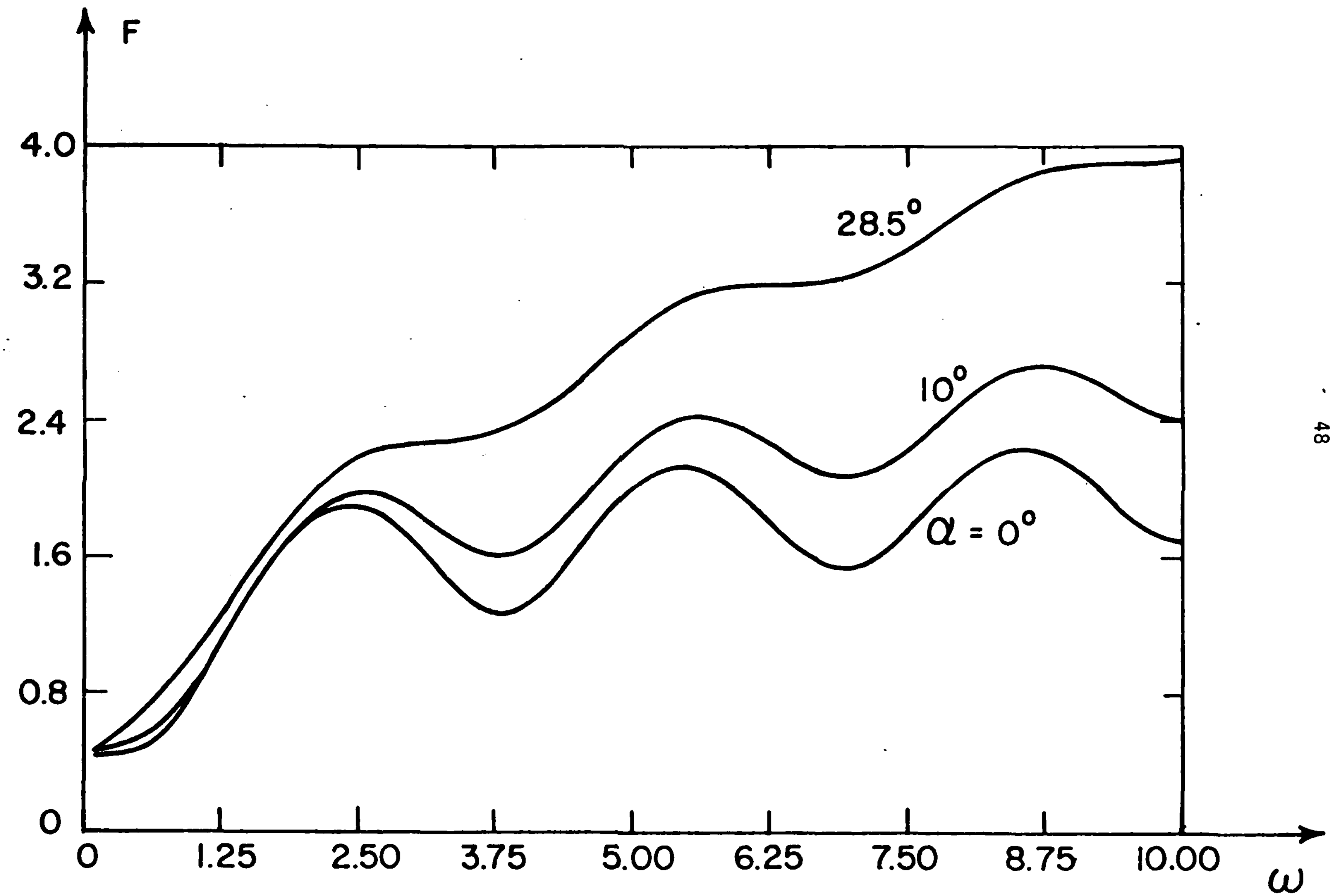


FIGURE 18. THE MAGNITUDE OF THE RATIO OF THE UNSTEADY LIFT FOR LIFTING AIRFOILS TO THE SEARS FUNCTION VERSUS THE REDUCED FREQUENCY.

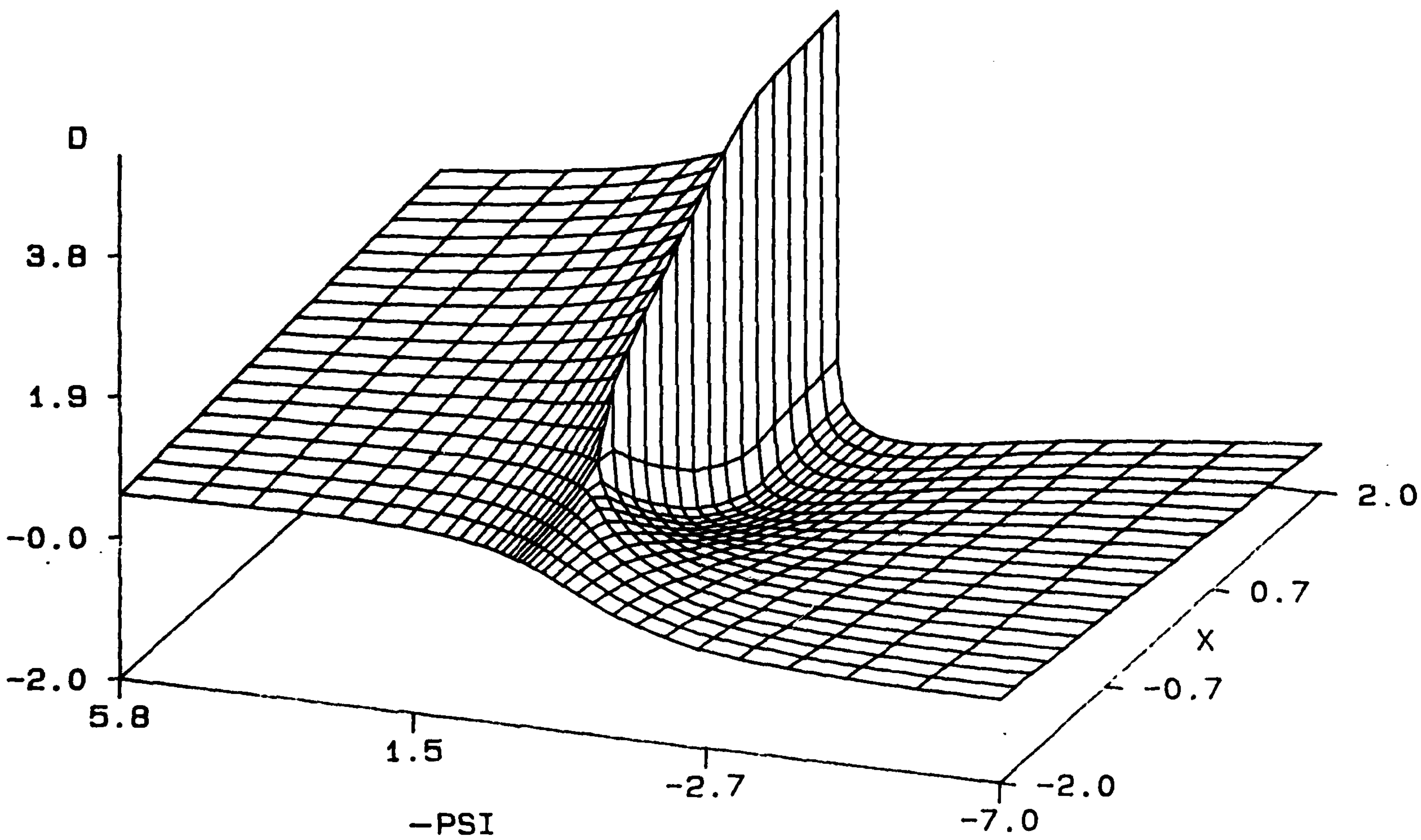


FIGURE 19. THE DRIFT FUNCTION MINUS POTENTIAL FUNCTION
JOUKOWSKI AIRFOIL (CAMBER=0.1 THICKNESS=0.1 INCIDENCE=10 DEG)

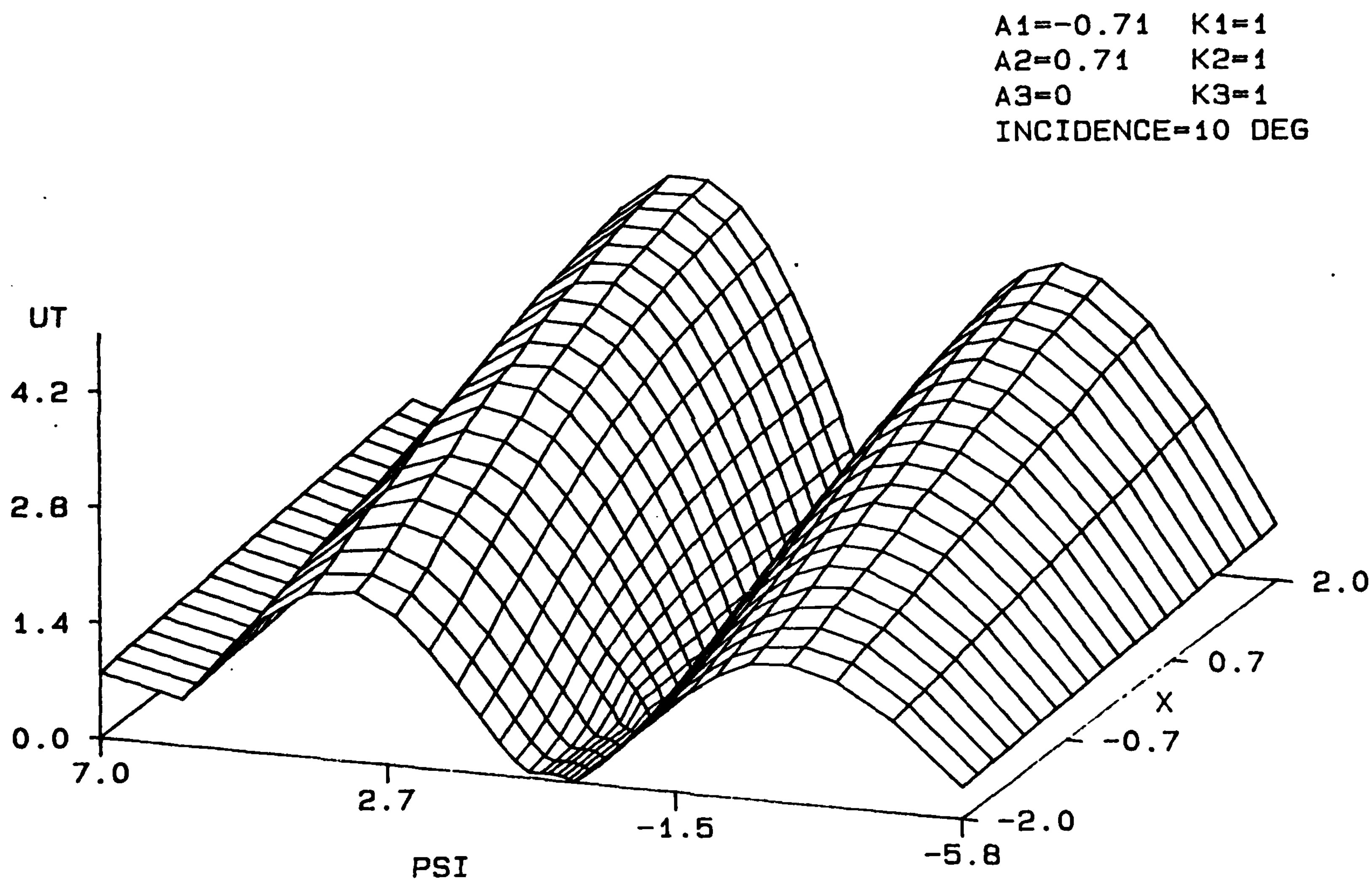


FIGURE 20. THE MAGNITUDE OF THE STREAMWISE VELOCITY (HA-METHOD) JOUKOWSKI AIRFOIL (CAMBER=0.1 THICKNESS=0.1)

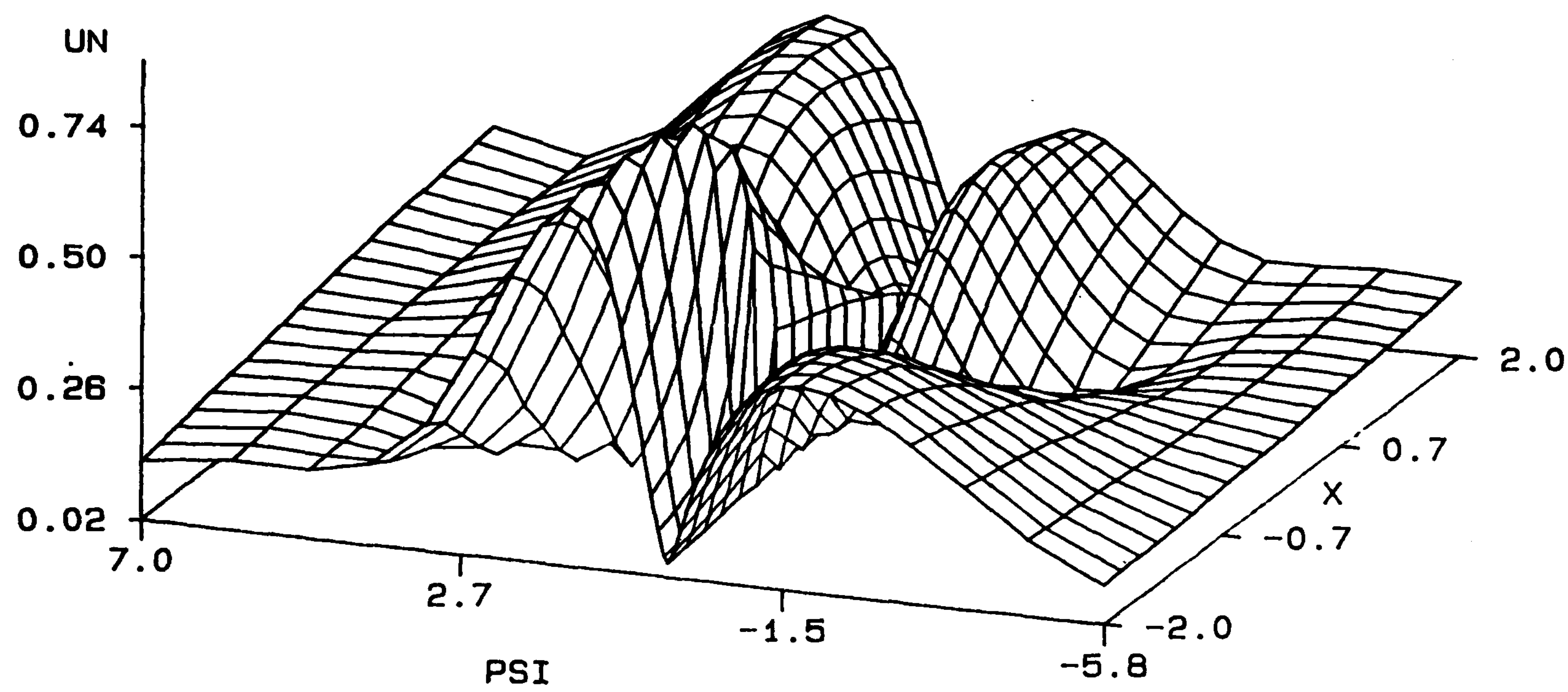


FIGURE 21. THE MAGNITUDE OF THE NORMAL VELOCITY (HA-METHOD) JOUKOWSKI AIRFOIL (CAMBER=0.1 THICKNESS=0.1)

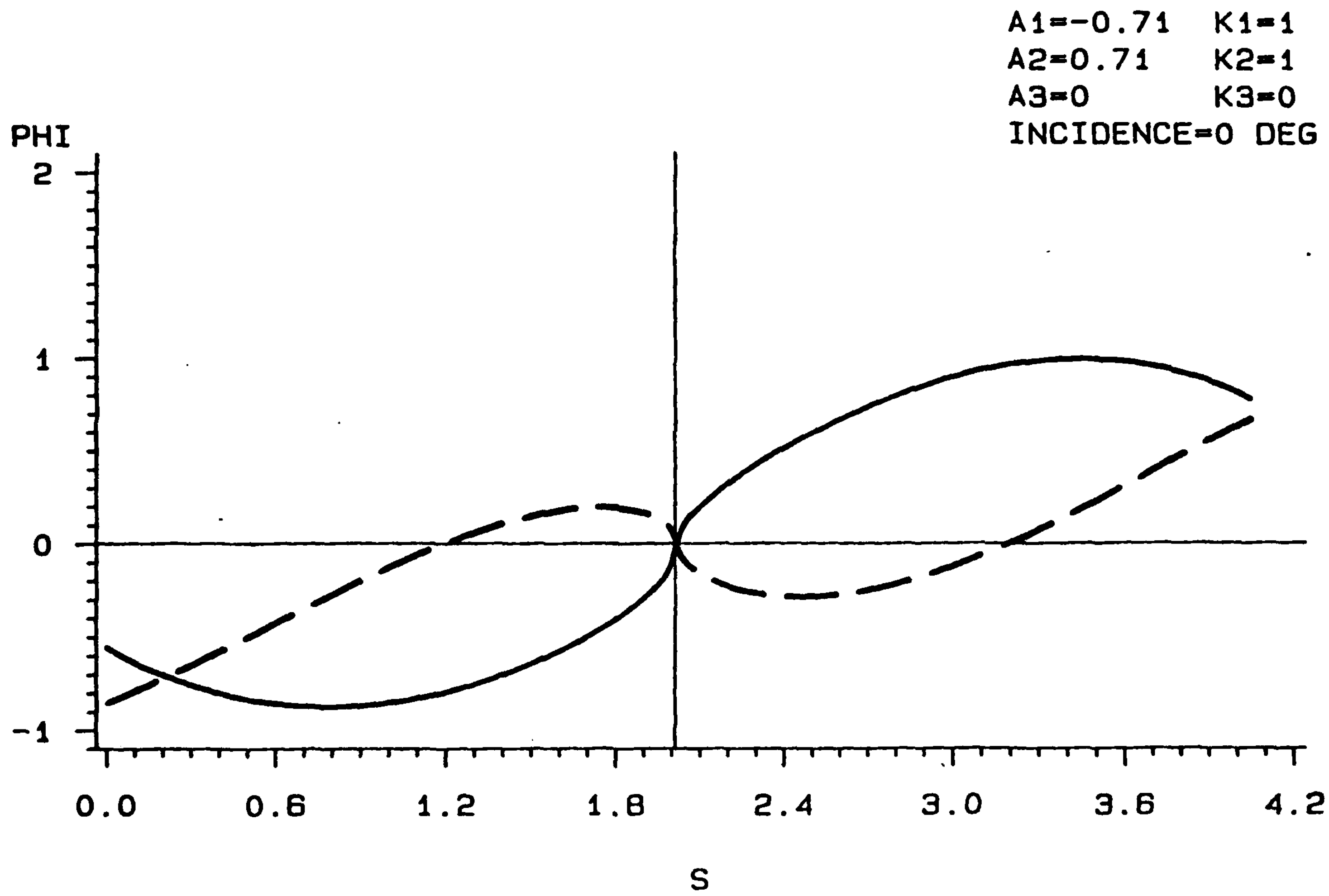


FIGURE 22. POTENTIAL FUNCTION: REAL (SOLID LINE) AND IMAGINARY (DASHED LINE)
JOUKOWSKI AIRFOIL: CAMBER=0.05 THICKNESS=0.05

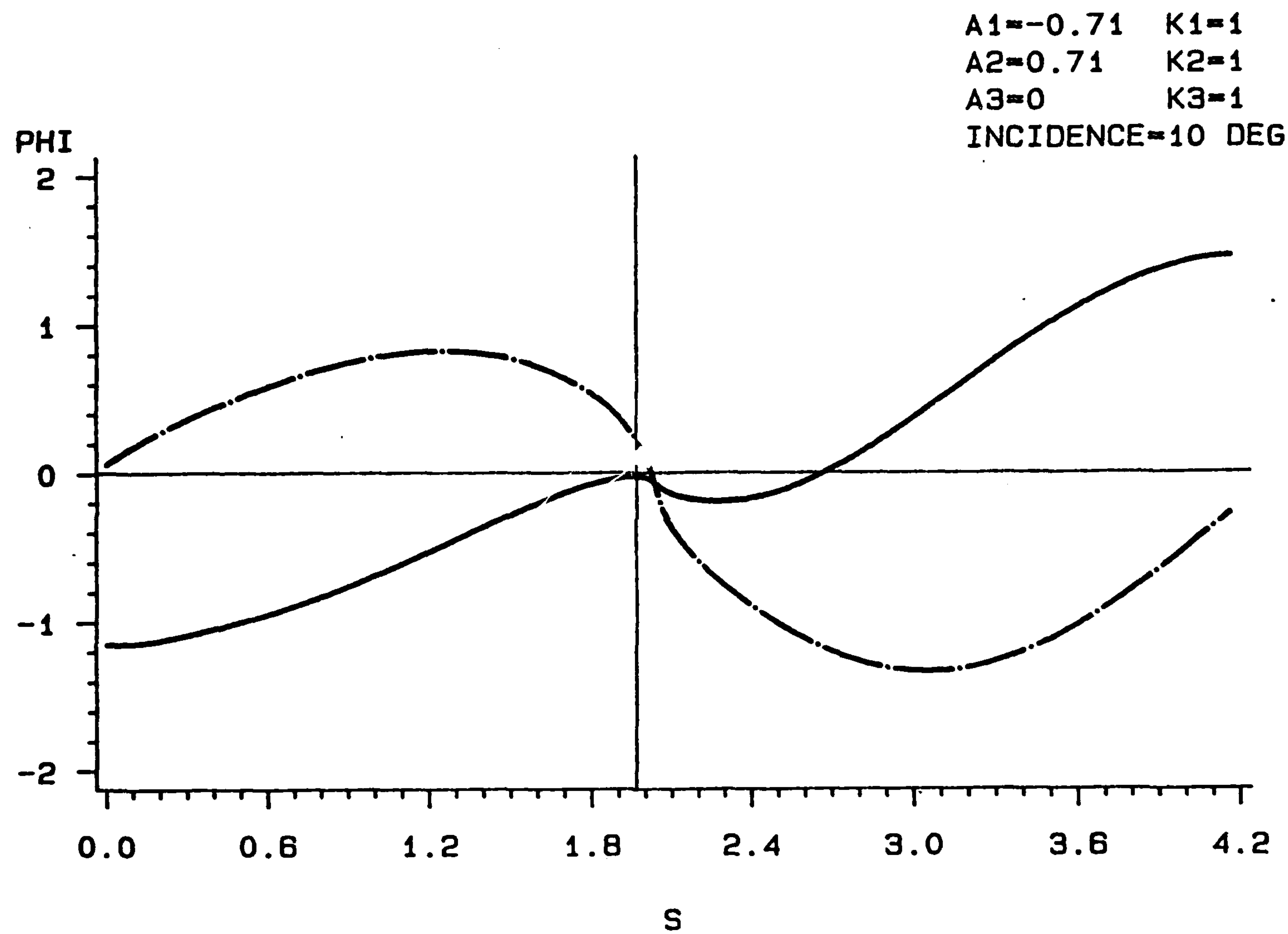


FIGURE 23. POTENTIAL FUNCTION: REAL (SOLID LINE) AND IMAGINARY (DASHED LINE)
JOUKOWSKI AIRFOIL: CAMBER=0.1 THICKNESS=0.1

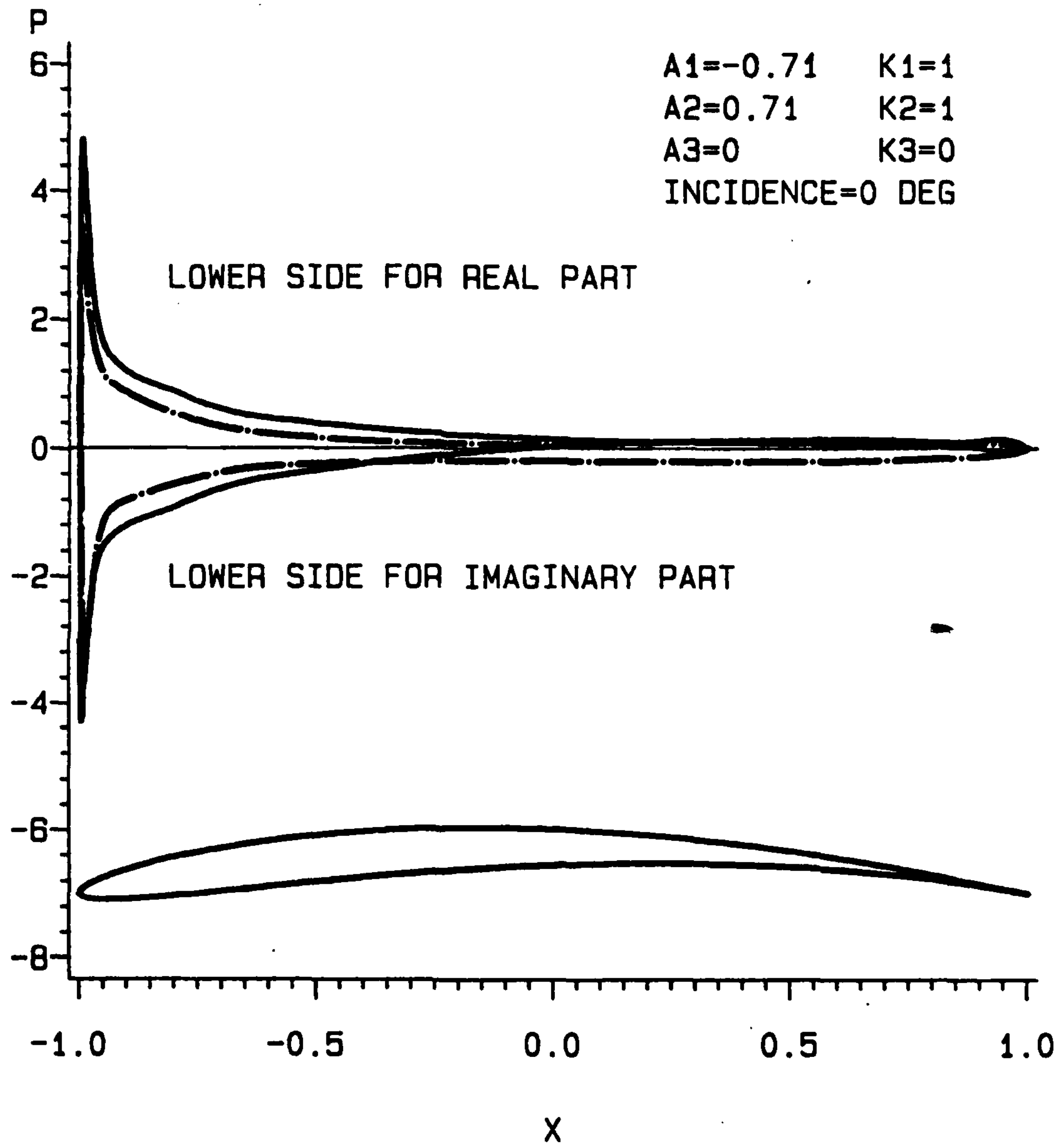


FIGURE 24. UNSTEADY PRESSURE
REAL (SOLID LINE) AND IMAGINARY (DASHED LINE)
JOUKOWSKI AIRFOIL: CAMBER=0.05 THICKNESS=0.05

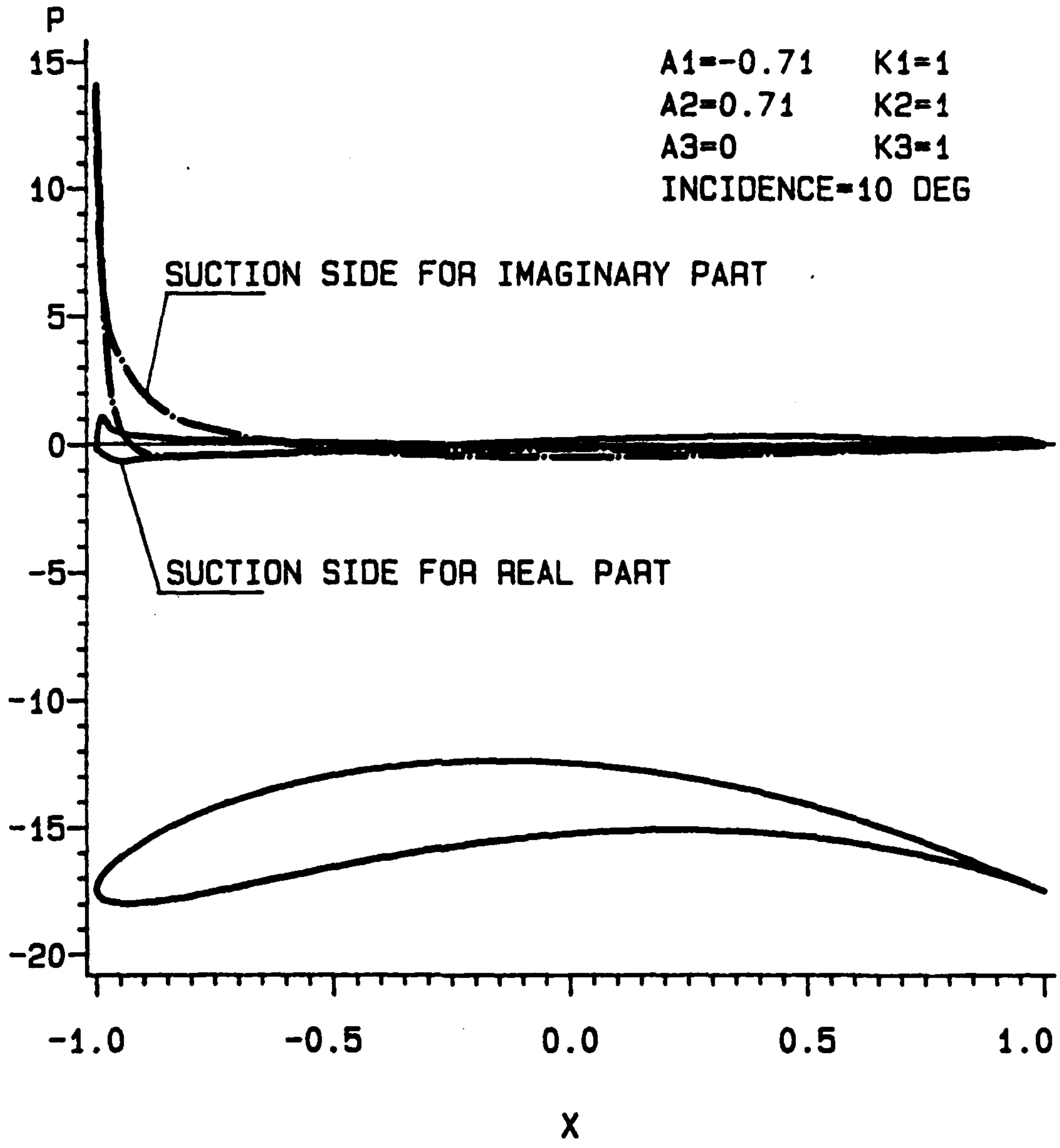


FIGURE 25. UNSTEADY PRESSURE
REAL (SOLID LINE) AND IMAGINARY (DASHED LINE)
JOUKOWSKI AIRFOIL: CAMBER=0.1 THICKNESS=0.1

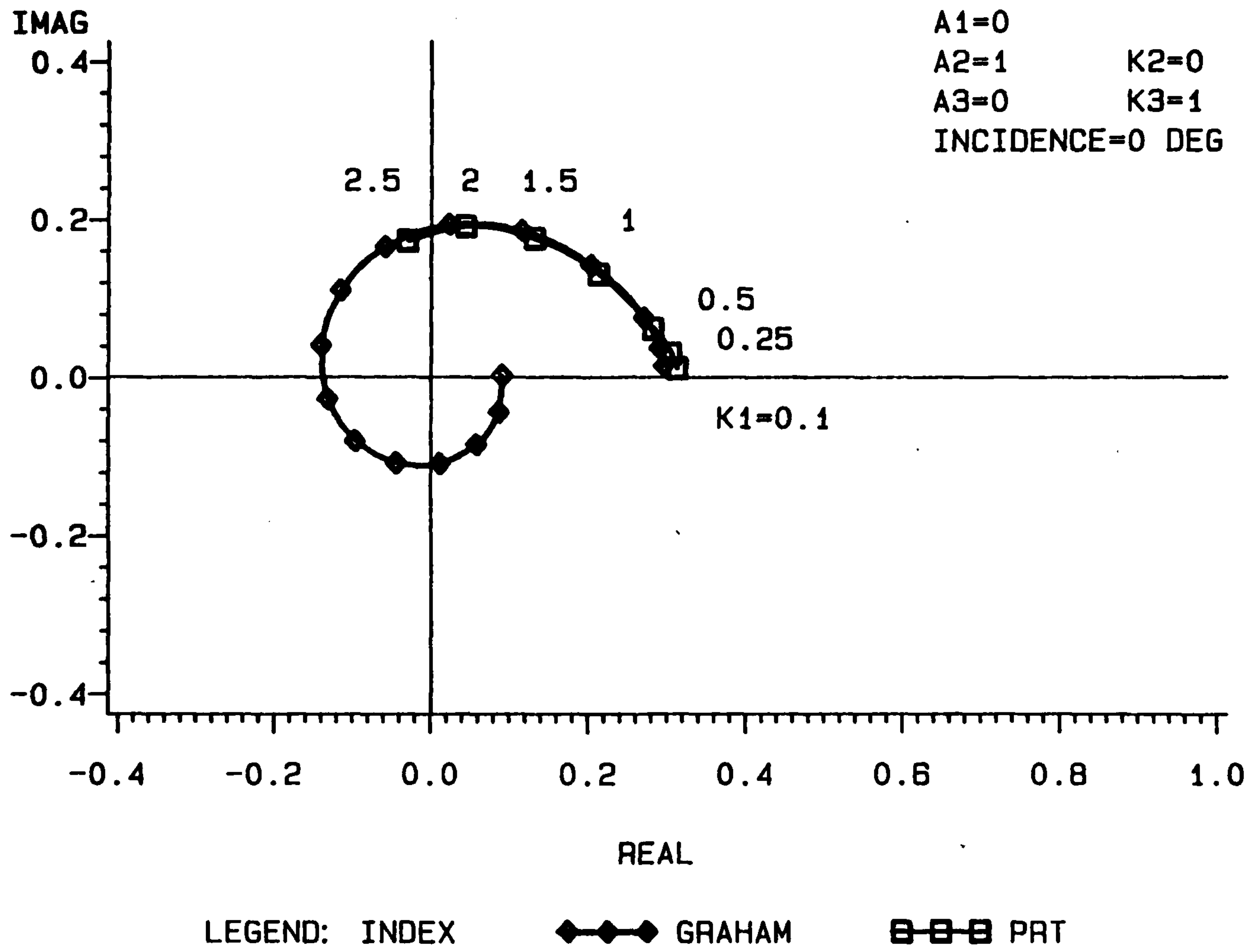


FIGURE 26. THE UNSTEADY LIFT COEFFICIENT
JOUKOWSKI AIRFOIL: CAMBER=0 THICKNESS=0.05

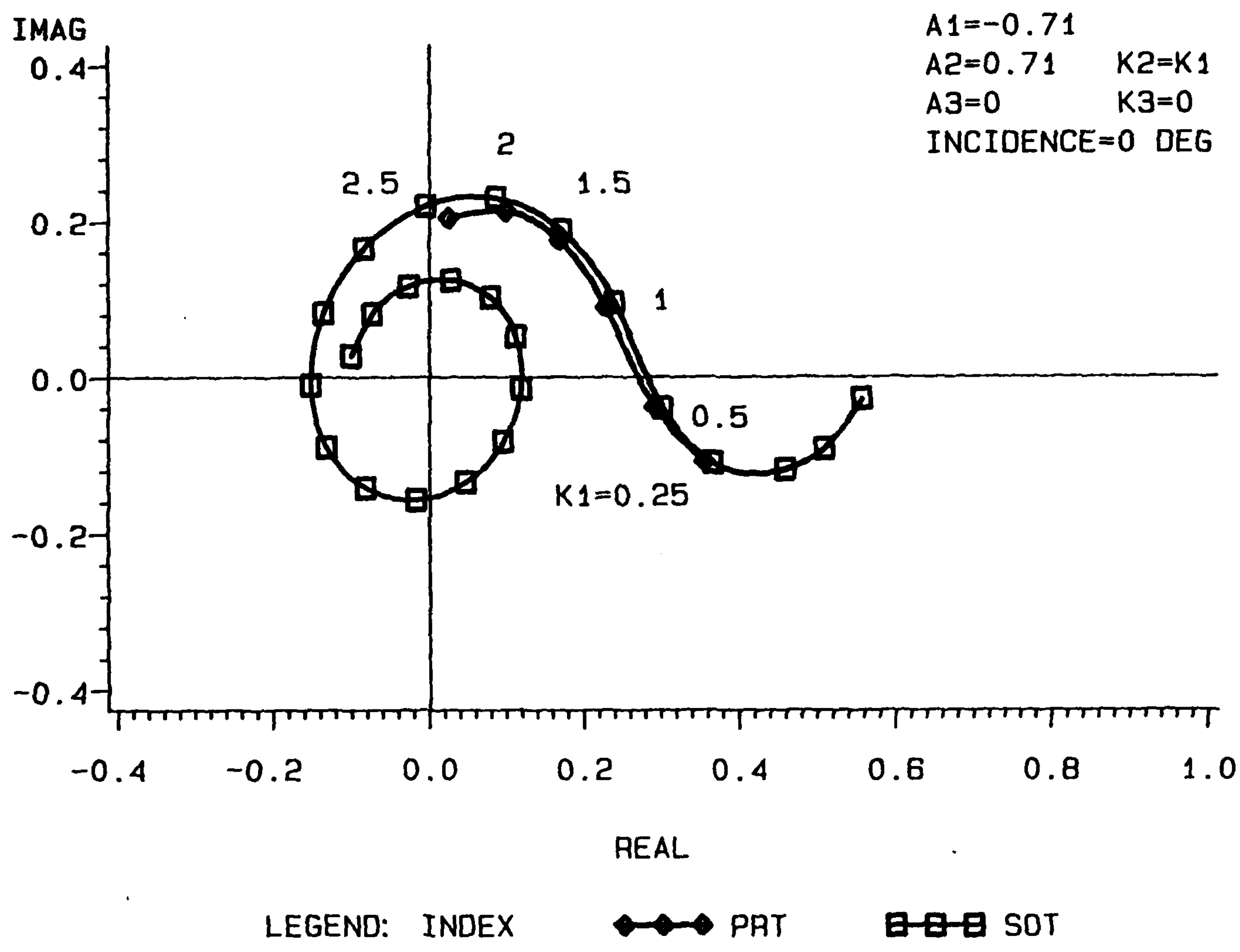


FIGURE 27. THE UNSTEADY LIFT COEFFICIENT
JOUKOWSKI AIRFOIL: CAMBER=0.05 THICKNESS=0.05

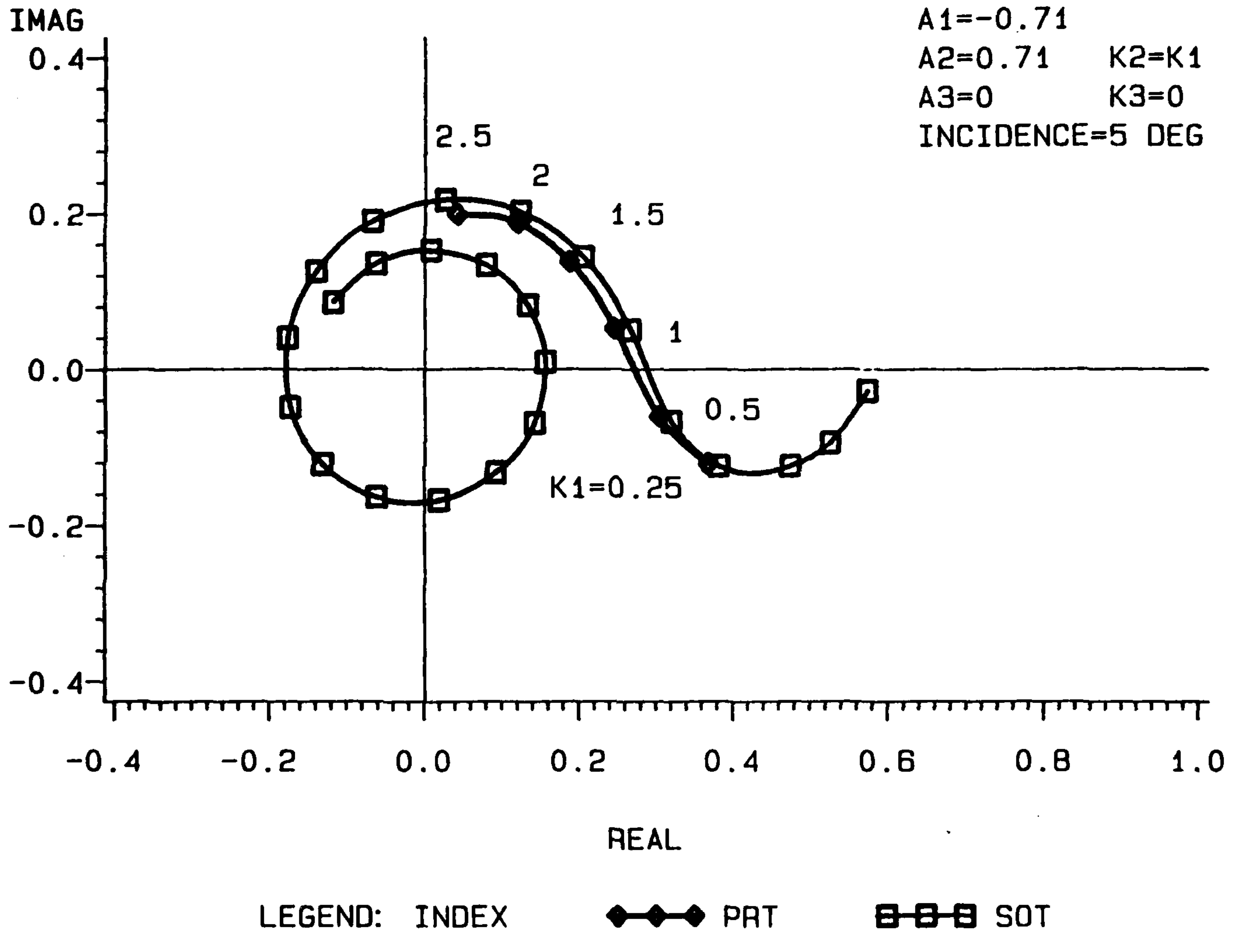


FIGURE 28. THE UNSTEADY LIFT COEFFICIENT
JOUKOWSKI AIRFOIL: CAMBER=0 THICKNESS=0.05

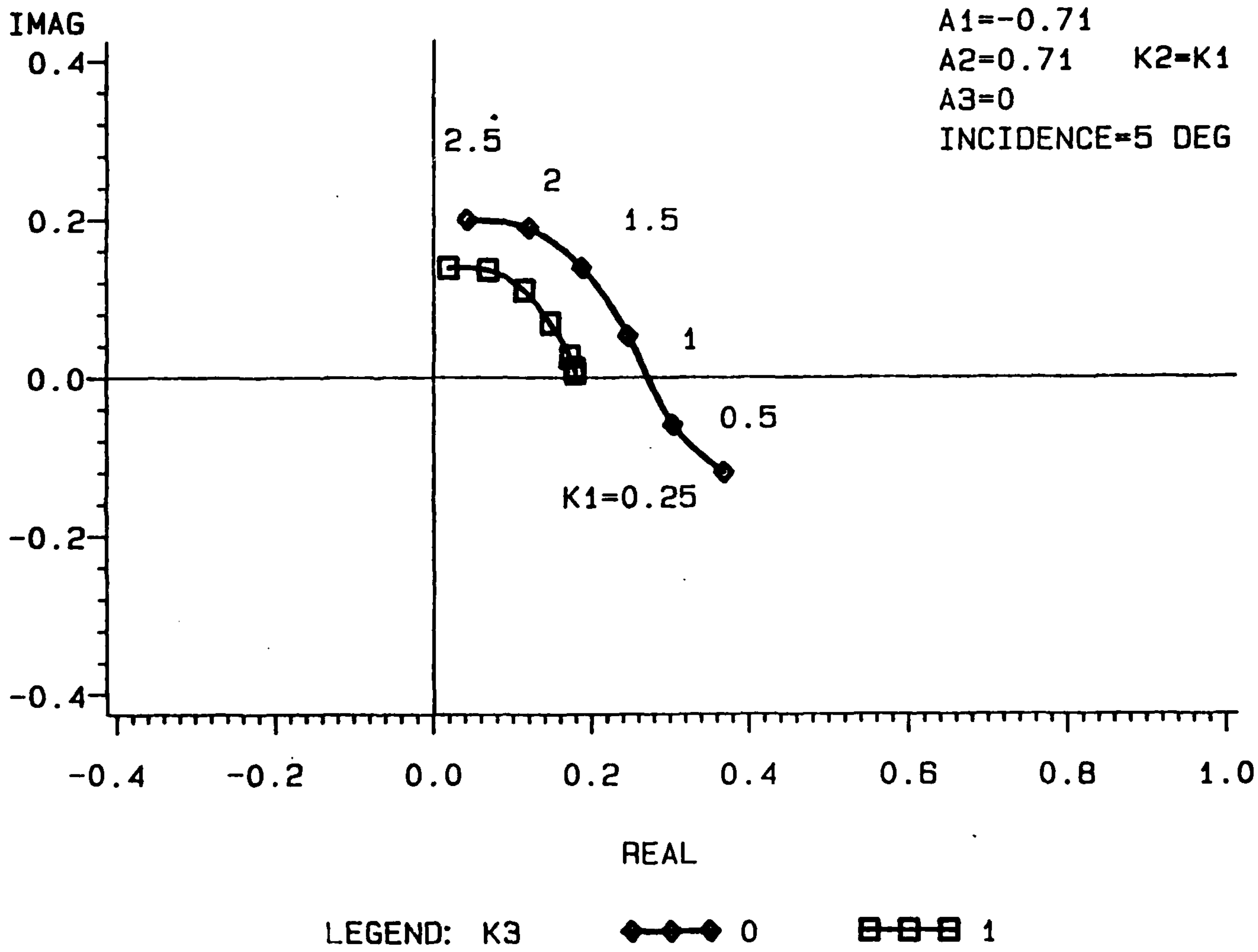


FIGURE 29. THE UNSTEADY LIFT COEFFICIENT
JOUKOWSKI AIRFOIL: CAMBER=0 THICKNESS=0.05

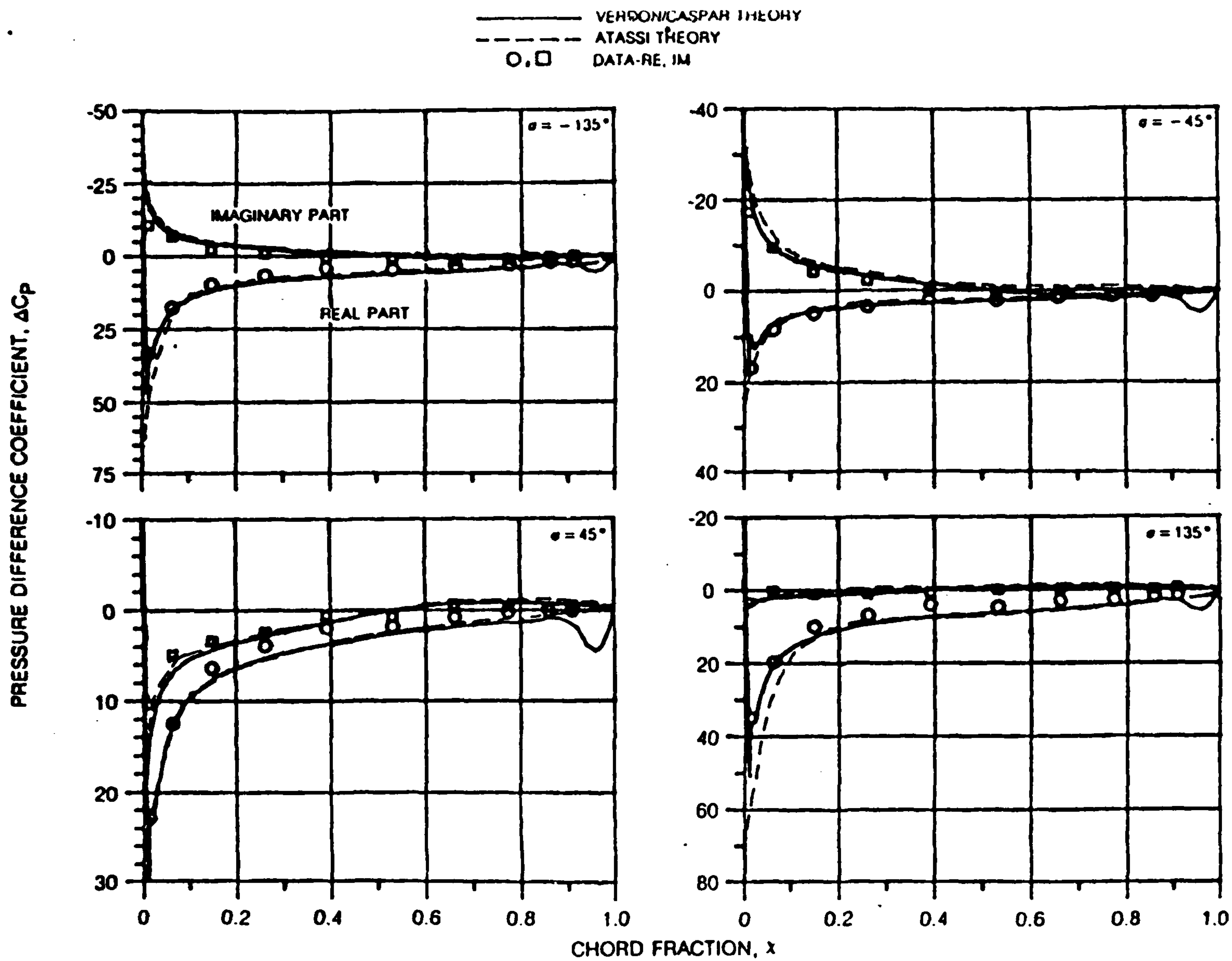


FIGURE 30. COMPARISON OF TWO THEORIES WITH EXPERIMENT FOR $\alpha = 2 \pm 0.5$ DEG AND $k = 0.122$.

VIII. PUBLICATIONS RESULTING FROM OUR WORK UNDER AFOSR SPONSORSHIP

All publications listed below are authored by Professor H. Atassi. Only co-authors are listed.

"Three-Dimensional Periodic Disturbances Acting Upon Airfoils in Cascade", Aeroelasticity in Turbomachines, Ed. P. Suter, Juris-Verlag Zurich, pp. 383-398, 1981.

"Stability and Flutter Analysis of Turbine Blades at Low Speed", Aeroelasticity in Turbomachines, Ed. P. Suter, Juris-Verlag Zurich, pp. 187-201, 1981. Co-author T.J. Akai.

In Preparation

"Regularization of Goldstein's Splitting of Unsteady Vortical and Entropic Distortions of Potential Flows," Invited paper, 19th Annual Meeting, Society of Engineering Science, October 27-29, 1982, Rolla, Missouri.

"A Uniformly Valid Splitting of Unsteady Vortical and Entropic Disturbances of Potential Flows."

"Three-Dimensional Periodic Vortical Distrubances Acting Upon an Airfoil," Co-author J. Grzedzinski

IX. INVITED LECTURES

These are lectures given by Professor H. Atassi on topics related to the present AFOSR grant.

"Unsteady Aerodynamics of Lifting Airfoils," NASA Lewis Research Center, Cleveland, Ohio, December 10, 1981.

"Unsteady Aerodynamics: Early and Recent Developments," Ohio State University, Columbus, Ohio, May 21, 1982.

X. PERSONNEL

All people who worked under the present grant are listed below:

Hafiz Atassi, Professor and Principal Investigator
John Grzedzinski, Research Assistant
Victor Nee, Professor (Partial Support)
Haider Raza, Research Assistant (Partial support)

END

DATE
FILMED

5 – 83

DTIC